BC Calculus
Curve Sketching & Theorems: Review

Name_______________________________Pd.___
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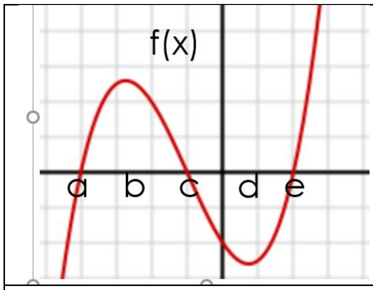
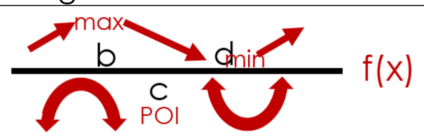
Please start off every review with reading your notecards for that unit several times!!!!! This is a very limited review!!!!

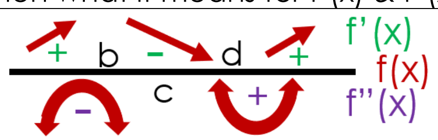
You should be able to look at a graph of f(x) and f'(x) and completely fill out a sign chart.

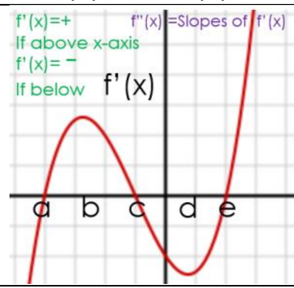


If you have f(x) fill out f(x) on sign line first.

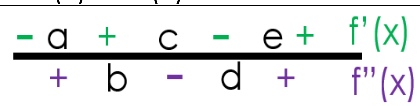


Then what it means for f'(x) & f''(x).

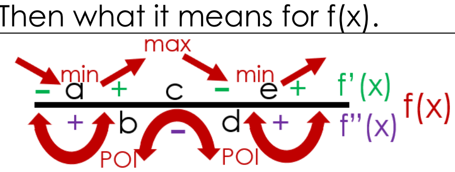




If you have f'(x) fill out signs of f'(x) & f''(x) first.



Then what it means for f(x).



<u>Critical Number</u>-f'(x)=0 or f'(x)=undefined

<u>Max</u>-f'(x) changes from pos to neg

<u>Min</u>-f'(x) changes from neg to pos

<u>Increasing</u>-f'(x)>0

<u>Decreasing</u>-f'(x)<0

<u>Point of Inflection</u>-f''(x)=0 or f''(x)=undefined **and** f''(x) changes signs

<u>Concave up</u>-f''(x)>0

<u>Concave down</u>-f''(x)<0

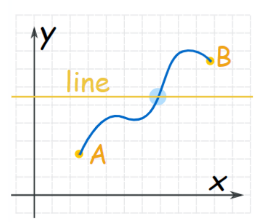
- Justify Max/Min

<u>First derivative test</u>-f'(x) changes sign

<u>Second derivative test</u>-
f'(x)=0/ud & f''(x)>0=min
f''(x)<0=max

<u>Intermediate Value Thm.</u>- **If** f(x) is continuous and you know 2 y-values **Then** f(x) goes through all the y-values in-between.

The idea behind the Intermediate Value Theorem is this:

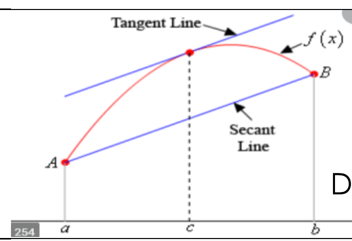


When we have **two points** connected by a continuous curve:

- one point below the line
- the other point above the line

... then there will be **at least one place** where the curve crosses the line!

<u>Mean Value Thm.</u> -**If** f(x) is continuous and differentiable **Then** the average rate of change=the instant rate of change



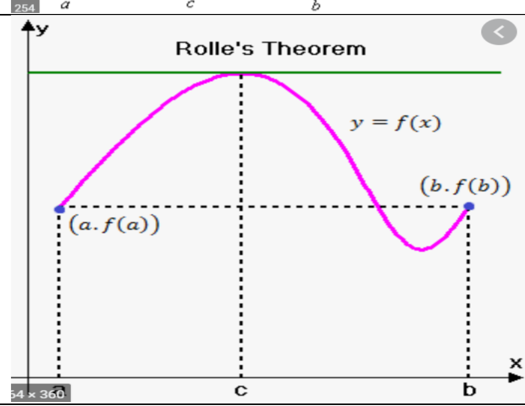
$$f'(x) = \frac{f(b)-f(a)}{b-a}$$

Instant=Average

Derivaive=Slope

If your average velocity if 45mph on the way to school at some point you had to drive 45 mph

<u>Rolle's Thm</u>.-**If** f(x) is continuous and differentiable and f(a)=f(b) **Then** somewhere on (a,b) f'(x)=0 (max or min) at least 1 time.



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1. Find the critical numbers of $f(x) = x^3 - 12x^2$.

a.) 0 & 8
b.) 3 & 8
c.) $-8, 0, \& 3$
d.) 1

2. Given that $f(x) = -x^2 + 12x - 28$ has a relative maximum at x=6, choose the correct statement.

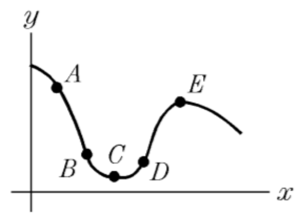
a.) f' is negative on the interval $(-\infty, 6)$
b.) f' is negative on the interval $(-\infty, \infty)$
c.) f' is negative on the interval $(6, \infty)$
d.) f' is negative for all real values

3. Over which interval(s) is $f(x) = \dfrac{x^2}{x^2 + 4}$ increasing?

a.) $(0, \infty)$
b.) $(-\infty, 0)$
c.) $(-\infty, \infty)$
d.) $(-\infty, 0) \& (2, \infty)$

4. At which of the five points shown on the graph of f(x) are the graphs of $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ both positive?

a.) A
b.) B
c.) D
d.) E



5. If $f(x) = x^3 - 3x^2 - x + 7$, determine its point of inflection.

a.) $(1, 4)$
b.) $(2, 1)$
c.) $(3, 4)$
d.) $(-1, 4)$

6. Let f be defined by $f(x) = x^2(x-3)$ for all real numbers x. For what values of x is the function increasing?

a.) $0 < x < 2$
b.) $0 < x < 3$
c.) $x > 0$
d.) $x < 0$ & $x > 2$

7. Let $f''(x) = 3x^2 - 4$ and let f(x) have critical numbers $-2, 0, \& 2$. Use the 2nd derivative test to determine which critical numbers, if any, gives a relative maximum.

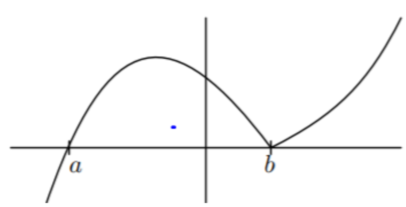
a.) -2
b.) 2
c.) 0
d.) $-2, 0, \& 2$

8. The Mean Value Theorem does not apply to f(x) = |x − 3| on [1, 4] because

a.) f(x) is not continuous on [1, 4]
b.) (x) is not differentiable on (1, 4)
c.) f(1) ≠ f(4)
d.) f(1) > f(4)
e.) None of these

9. The function F below satisfies the conclusion of Rolle's Theorem in the interval [a, b] because

a.) F is continuous on [a, b]
b.) F is differentiable on (a, b)
c.) F(a) = F(b)=0
d.) All three statements A, B and C
e.) None of these



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10. The graph of f', the derivative of f, is shown in the figure for $-2 \le x \le 5$. On which intervals is f increasing?

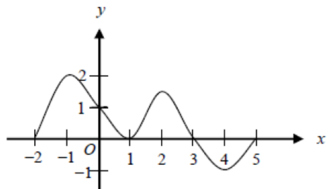
a.) $[-2,1]$

b.) $[-2,3]$

c.) $[3,5]$

d.) $[0,1.5]$ & $[3,5]$

e.) $[-2,-1],[1,2],\&[4,5]$



Graph of f'

11. If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

a.) $f'(c) = \dfrac{f(b)-f(a)}{b-a}$ for some c such that $a < c < b$

b.) $f'(c) = 0$ for some c, such that $a < c < b$

c.) f has a minimum value on $a \le c \le b$

d.) f has a maximum value on $a \le c \le b$

e.) $\int_a^b f(x)dx$ exists

12. The graph of the derivative of a function f is shown in the figure. At which of the following values of x does f have a relative maximum?

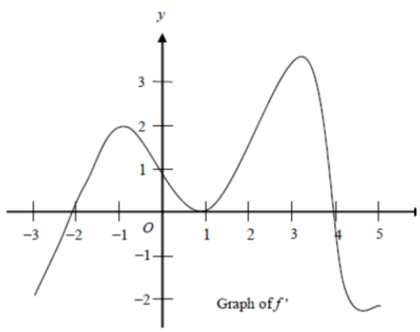
a.) -2

b.) 1

c.) 4

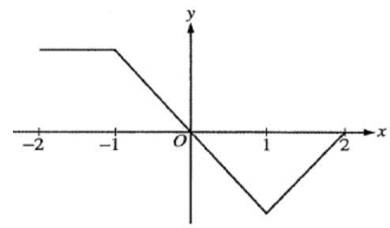
d.) $-1 \& 3$

e.) $-2, 1, \& 4$



Graph of f'

13. The graph of f', the derivative of the function f, is shown in the figure. Which of the following statements is true about f?



Graph of f'

a.) f is decreasing for $-1 \le x \le 1$.

b.) f is increasing for $-2 \le x \le 0$.

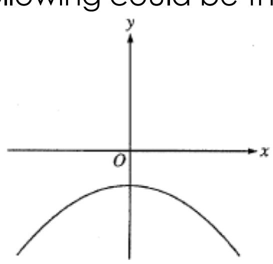
c.) f is increasing for $1 \le x \le 2$.

d.) f has a local minimum at x=0.

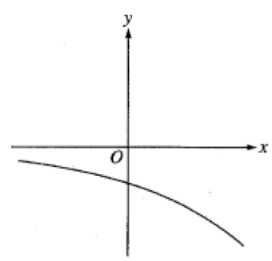
e.) f is not differentiable at $x = -1$ & $x = 1$

14. The function f has the property that f(x), f'(x), & f''(x) are negative for all real values x. Which of the following could be the graph of f?

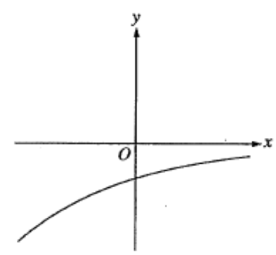
a.)



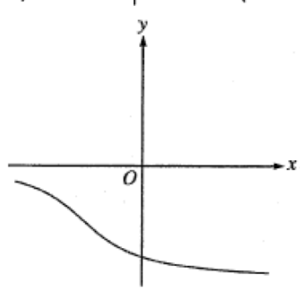
b.)



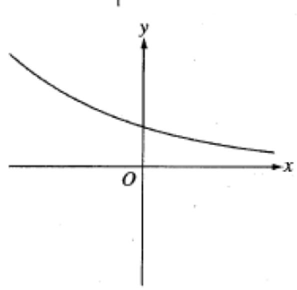
c.)



d.)



e.)



15. The function f defined by $f(x) = 4x^2 - 5x + 1$. The application of the Mean Value Theorem to f on the interval $0 < x < 2$ guarantees the existence of a value c, where $0 < c < 2$ such that f'(c)=

a.) 1   b.) 3   c.) 7   d.) 8

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Calculators may be used on this section:

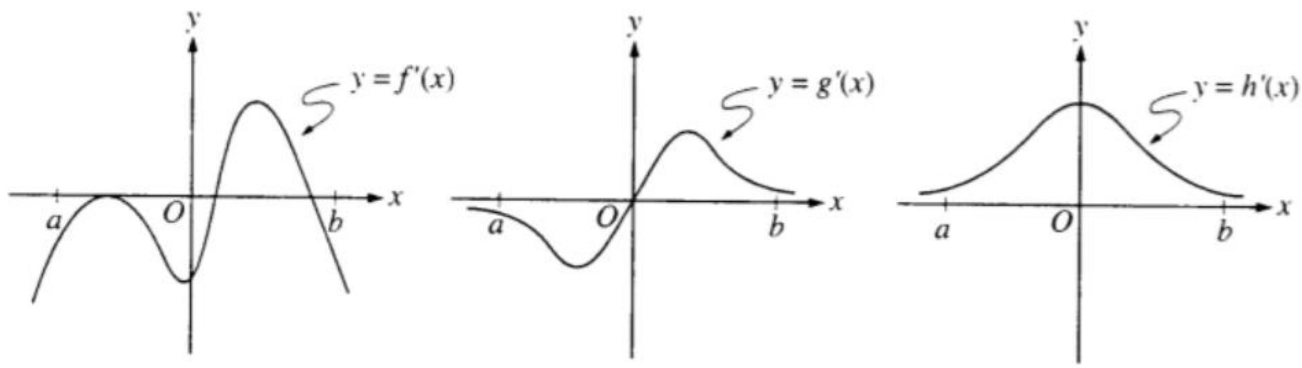
16. Let f be the function with derivative given by $f'(x) = \sin(x^2 + 1)$. How many relative extrema does f have on the interval $2 < x < 4$?

a.) one
b.) two
c.) three
d.) four
e.) five

17. The function f has first derivative given by $f'(x) = \dfrac{\sqrt{x}}{1 + x + x^3}$. What is the x-coordinate of the inflection point of the graph of f?

a.) 1.008
b.) 0.473
c.) 0
d.) -0.278
e.) none

18.



The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?

a.) f
b.) g
c.) h
d.) f & g
e.) f, g, & h

19. Let f be the function given by $f(x) = \cos(2x) + \ln(3x)$. What is the least value of x
at which the graph of f changes concavity?

a.) 0.56
b.) 0.93
c.) 1.18
d.) 2.38
e.) 2.44

20. If the derivative of f is given by $f'(x) = e^x - 3x^2$, at which of the following values of x does f have a relative maximum value?

a.) -0.46
b.) 0.20
c.) 0.91
d.) 0.95
e.) 3.73

21. The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at x=

a.) -1.58
b.) -163
c.) -1.67
d.) -1.89
e.) -2.33

22. What are all the values of x for which the function f defined by $f(x) = x^3 + 3x^2 - 9x + 7$ is increasing?

a.) $-3 < x < 1$
b.) $-1 < x < 1$
c.) $x < -3$ & $x > 1$
d.) $x < -1$ & $x > 3$
e.) All real numbers