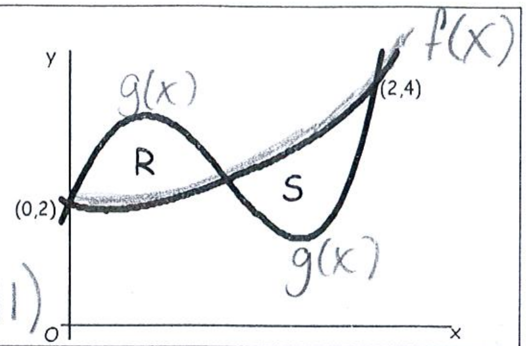
Calculator

Area/Volume #1

Name________________________

Date_______________Pd.___

Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ (y_1) and $g(x) = x^4 - 6.5x^2 + 6x + 2$ (y_2) let R and S be the two regions enclosed by the graphs of f and g shown in the figure to the right.



Intersection (1.0328, 2.4011)

a.) Find the sum of the areas of regions R and S.

R + S

$$\int_0^{1.0328} g(x) - f(x)\,dx + \int_{1.0328}^{2} f(x) - g(x)\,dx$$

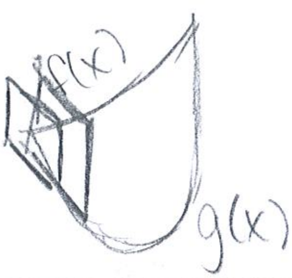
.99742 + 1.0069

2.004

1: limits
2: Integrands
1: Answer

b.) Region S is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

dx



$\int_{x_1}^{x_2}$ Area square dx side = top - bottom

$\int (\text{side})^2 dx$

$\int_{1.0328}^{2} [f(x) - g(x)]^2 dx = 1.283$ +2: Integrand +1: Answer

c.) Let h be the vertical distance between the graphs of f and g in region S. Find the rate of which h changes with respect to x when $x = 1.8$.

Find the Rate of which h changes: Find $\frac{dh}{dx}$

$h = f(x) - g(x)$

$h = \cancel{1} + x + e^{x^2-2x} - x^4 + 6.5x^2 - 6x - 2$

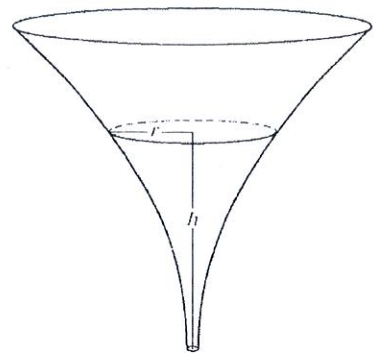
$\frac{dh}{dx} = 0 + 1 + (2x-2)e^{x^2-2x} - 4x^3 + 13x - 6$ +1: Considers h'

$\left.\frac{dh}{dx}\right|_{x=1.8} = -3.811$ OR -3.812 +1: Answer

Non-Calculator
Area/Volume #5

Name____________________
Date______________Pd_____

The inside of a funnel of height 10 inches has a circular cross sections, as shown in the figure to the right. At height h, the radius of the funnel is given by $r = \frac{1}{20}(3 + h^2)$, where $0 \le h \le 10$. The units of r and h are inches.



a.) Find the average value of the radius of the funnel.

$r = \frac{1}{20}(3+h^2)$ $\text{Avg val} = \frac{1}{10-0}\int_0^{10} \frac{1}{20}(3+h^2)\,dh$ +1: integral

$\frac{1}{10}\cdot\frac{1}{20}\left[3h+\frac{h^3}{3}\right]_0^{10} = \frac{1}{200}\left[30+\frac{1000}{3}-0-0\right] = \frac{1}{200}\left[\frac{90}{3}+\frac{1000}{3}\right] = \frac{1090}{200}$

+1: antiderivative

+1: answer

b.) Find the volume of the funnel.

$V = \pi\int_0^{10} R^2 = \pi\int_0^{10}\left[\frac{1}{20}(3+h^2)\right]^2 dh = \frac{\pi}{400}\int_0^{10} 9+6h^2+h^4\,dh$

$\frac{\pi}{400}\left[9h+\frac{\cancel{6}h^3}{\cancel{3}}+\frac{h^5}{5}\right]_0^{10} = \frac{\pi}{400}\left[90+2(1000)+\frac{\cancel{100000}}{\cancel{5}}-0-0-0\right]$ +1: integrand

20000
2000
90

$\frac{\pi}{20}\left[90+2000+20000\right] = \frac{22090}{400}\pi = \frac{2209}{40}\pi$ +1: antiderivative

+1: answer

c.) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is h=3 inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

Know: $\frac{dr}{dt} = -\frac{1}{5}\frac{\text{in}}{\text{sec}}$

Find: $\frac{dh}{dt} =$

When: $h = 3\text{in}$

Eqn: $r = \frac{1}{20}(3+h^2)$

Der: $\frac{dr}{dt} = \frac{1}{20}\left[0+2h\frac{dh}{dt}\right]$ +2: Chain Rule

$20\frac{dR}{dt} = 2h\frac{dh}{dt}$

$20\left(-\frac{1}{5}\frac{\text{in}}{\text{sec}}\right) = 2(3\text{in})\frac{dh}{dt}$

$-4 = 6\frac{dh}{dt}$

$\frac{dh}{dt} = -\frac{2}{3}\frac{\text{in}}{\text{sec}}$ +1: answer