

Please start off every review with reading your notecards for that unit several times!!!! This is a very limited review!!!!

<p><u>Integration</u>-finding the area bounded between a function & the x-axis. Area above the x-axis is</p> <p>and area below the x-axis is</p>	
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$\int \text{rate} =$	$\int \frac{\text{miles}}{\text{hour}} =$	$\int \frac{\text{gallon}}{\text{min}} =$	$\int \frac{\text{lbs of bananas}}{\text{hour}} =$
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Average Value- $\frac{1}{hr_2 - hr_1} \int_{hr_1}^{hr_2} \frac{\text{miles}}{\text{hour}}$

Estimating Area-Right endpoint, left endpoint, midpoint & Trapezoidal approximation. We practiced from a graph, function, & table

Right-area rectangle $A = \text{width} \cdot \text{length}(f(x))$	Left-area rectangle $A = \text{width} \cdot \text{length}(f(x))$	Mid=area rectangle $A = \text{width} \cdot \text{length}(f(x))$	Trap=area trapezoid $A = \frac{\text{height}}{2} (b_1 + b_2)$
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Fundamental Theorem of Calculus- $\frac{d}{dx} \int_{\#}^x f(t)dt = f(x)$ and $\frac{d}{dx} \int_{\#}^{AT} f(t)dt = f(AT) \cdot AT'$

• Area & Volume are never negative

<p><u>Area</u>- $\int_{x_1}^{x_2} \text{top function} - \text{bottom function} dx$ or $\int_{y_1}^{y_2} \text{right function} - \text{left function} dx$ You chose dx or dy</p>	
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<p><u>Volume</u>(cross sections perpendicular to)- \perp -to x - axis = $\int_{x_1}^{x_2} \text{area of cross section}$ \perp -to y - axis = $\int_{y_1}^{y_2} \text{area of cross section}$ You have no choice $\perp(x) = dx$ & $\perp(y) = dy$</p>	
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<p><u>Volume</u>-rotated about axis (solid hunka hunka) rotated about x - axis = $\pi \int_{x_1}^{x_2} R^2 dx$ (cross sections O's) or parallel to x (y=#) rotated about y - axis = $\pi \int_{y_1}^{y_2} R^2 dx$ or parallel to y (x=#) You have no choice x or to x=dx or y or to y=dy</p>	
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Volume-rotated about axis (hole)

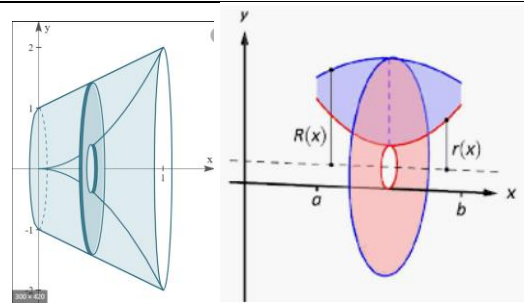
rotated about x – axis = $\pi \int_{x_1}^{x_2} R^2 - r^2 dx$

or parallel to x (y=#)

rotated about y – axis = $\pi \int_{y_1}^{y_2} R^2 - r^2 dy$

or parallel to y (x=#)

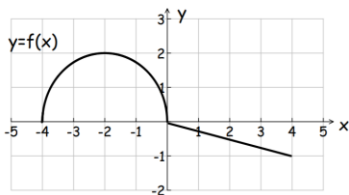
You have no choice x or || to x=dx or y or to || y=dy



Practice:

1. $\int_{-4}^4 f(x)dx =$

- a.) $2\pi + 1$
- b.) $2(\pi - 1)$
- c.) $\pi + 2$
- d.) $2(2\pi + 1)$
- e.) $\pi + 1$

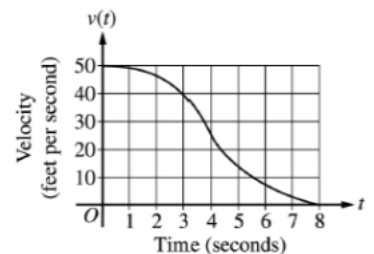


2. What is the average value of the function $g(x) = (2x + 3)^2$ on the interval $(-3, -1)$?

- a.) $\frac{7}{3}$
- b.) -4
- c.) 5
- d.) $\frac{14}{3}$
- e.) 3

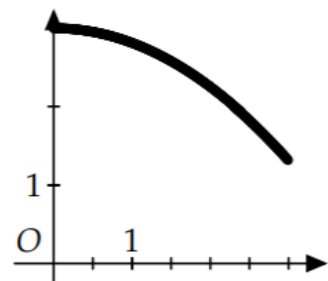
3. The graph gives the velocity, v in ft/sec, of a car for $0 \leq t \leq 8$, where t is the time in seconds. Of the following, which is the best estimate of the distance traveled by the car from $t = 0$ until the car comes to a complete stop?

- a.) 21 ft b.) 26 ft c.) 180 ft
- d.) 210 ft e.) 260 ft



4. The graph of the function f is shown for $0 \leq x \leq 3$. Of the following, which has the smallest value?

- a.) $\int_1^3 f(x)dx$
- b.) Left Riemann sum approximation of $\int_1^3 f(x)dx$ of equal length.
- c.) Right Riemann sum approximation of $\int_1^3 f(x)dx$ of equal length.
- d.) Midpoint Riemann sum approximation of $\int_1^3 f(x)dx$ of equal length.
- e.) Trapezoidal sum approximation of $\int_1^3 f(x)dx$ of equal length.



5. $\frac{d}{dx} \int_2^x \sqrt{1+t^2} dt =$

- a.) $\frac{x}{\sqrt{1+x^2}}$
- b.) $\sqrt{1+x^2} - 5$
- c.) $\sqrt{1+x^2}$
- d.) $\frac{x}{\sqrt{1+x^2}} - \frac{1}{\sqrt{5}}$
- e.) $\frac{x}{2\sqrt{1+x^2}} - \frac{1}{2\sqrt{5}}$

6. $F(x) = \int_1^{x^2} \sqrt{1+t^3} dt$, then $F'(x) =$

- a.) $2x\sqrt{1+x^6}$
- b.) $2x\sqrt{1+x^3}$
- c.) $\sqrt{1+x^6}$
- d.) $\sqrt{1+x^3}$

Application of Integration: Review

7. **Non-Calculator** What is the area of the region in the first quadrant bounded by the

graph of $y = e^{\frac{x}{2}}$ and the line $x = 2$?

- a.) $2e - 2$
- b.) $2e$
- c.) $\frac{e}{2} - 1$
- d.) $\frac{e - 1}{2}$
- e.) $e - 1$

8. **Calculator** What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$?

- a.) 10.667
- b.) 11.833
- c.) 14.583
- d.) 21.333
- e.) 32

9. **Calculator** Let R be the region in the first and second quadrants bounded above by the

graph of $y = \frac{20}{1+x^2}$ and below by the horizontal

line $y = 2$. R is the base of a solid whose cross sections perpendicular to the x-axis are semicircles. What is the volume of the solid?

- a.) 29.815
- b.) 174.268
- c.) 348.537
- d.) 443.771

10. **Non-Calculator** Let r be the region in the first quadrant bounded above by the graph of

$y = \sqrt{x}$ and below by the graph of $y = x^2$. R is

the base of a solid whose cross sections perpendicular to the y-axis are squares. Which of the following gives the volume to the solid?

- a.) $\int_0^1 (\sqrt{x} - x^2)^2 dx$
- b.) $\int_0^1 (x - x^4)^2 dx$
- c.) $\int_0^1 (\sqrt{y} - y^2)^2 dy$
- d.) $\int_0^1 (\sqrt{y} - y^2) dy$

11. **Non-Calculator** Which integral below gives the volume of the solid of revolution obtained by rotating the bounded region between

$y = \sqrt{x}$ and $y = x^2$ about the line $y = 3$?

- a.) $\pi \int_0^1 (9 - x) - (9 - x^4)$
- b.) $\int_0^1 (3 - \sqrt{x})^2 - (3 - x^2)^2$
- c.) $\pi \int_0^1 (3 - \sqrt{x})^2 - (3 - x^2)^2$
- d.) $\int_0^1 (3 - x^2)^2 - (3 - \sqrt{x})^2$
- e.) $\pi \int_0^1 (3 - x^2)^2 - (3 - \sqrt{x})^2$

12. **Non-Calculator** Find the volume of the solid of revolution obtained by rotating the region in

the xy-plane bounded by $y = x^3 + 1$, $x = 1$, and $y = 1$ about the y-axis.

- a.) $\frac{11\pi}{3}$
- b.) $\frac{4\pi}{13}$
- c.) $\frac{3\pi}{7}$
- d.) $\frac{2\pi}{5}$
- e.) $\frac{2\pi}{9}$

Application of Integration: Review

13. **Calculator** What is the average value of

$$y = \frac{\cos x}{x^2 + x + 2} \text{ on the closed interval } [-1, 3] ?$$

- a.) -0.085
- b.) 0.090
- c.) 0.183
- d.) 0.244
- e.) 0.732

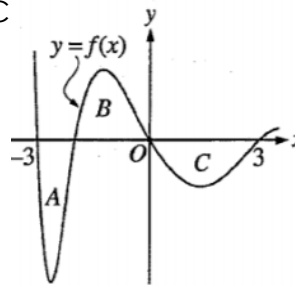
14. $\frac{d}{dx} \int_0^{x^2} \sin(t^3) dt =$

- a.) $-\cos(x^6)$
- b.) $\sin(x^3)$
- c.) $\sin(x^6)$
- d.) $2x \sin(x^3)$
- e.) $2x \sin(x^6)$

15. **N.C.** The regions A, B, & C in the figure are bounded by the graph of the function f and the x -axis. If the area of each region is 2, what is the value

$$\text{of } \int_{-3}^3 (f(x) + 1) dx ?$$

- a.) -2
- b.) -1
- c.) 4
- d.) 7
- e.) 12



16. **Calculator** The base of a solid is the region in the first quadrant bounded by the y -axis, the graph of $y = \tan^{-1}(x)$, the horizontal line $y = 3$, and the vertical line $x = 1$. For the solid, each cross section perpendicular to the x -axis is a square. What is the volume of the solid?

- a.) 2.561
- b.) 6.612
- c.) 8.046
- d.) 8.755
- e.) 20.773

17. **Non-Calculator** What is the area of the region between the graphs of $y = x^2$ & $y = -x$ from $x = 0$ to $x = 2$?

- a.) $\frac{2}{3}$
- b.) $\frac{8}{3}$
- c.) 4
- d.) $\frac{14}{3}$
- e.) $\frac{16}{3}$

18. **Non-Calculator** If the region enclosed by the y -axis, the line $y = 2$, and the curve $y = \sqrt{x}$ is revolved about the y -axis, the volume of the solid generated is

- a.) $\frac{32\pi}{5}$
- b.) $\frac{16\pi}{3}$
- c.) $\frac{16\pi}{5}$
- d.) $\frac{8\pi}{3}$
- e.) π