$\qquad$
$\qquad$ Day 3 \& 4 App Integration
$\int$ surface area of sphere $=\int 4 \pi r^{2}$

| Area of Cross Section $=$ |  |
| :--- | :--- |
| cross sections $\perp$ to x then side $=$ | cross sections $\perp$ to $y$ then side $=$ |
|  |  |

Cross section is Square
$\int$ Area of Square $=$


Cross Section is Isosceles Right Triangle
$\int$ Area of Isoceles $R t \Delta=$
Leg is side


Hypotenuse is side


Cross Section is Equilateral Triangle $\int$ Area of Equilateral $\Delta=$



Cross Section is Semicircle
$\int$ Area semi circle $=$
Radius is Side


Base: $y=-\frac{1}{3} x+4$
$y$-axis $(x=0)$
$x-\operatorname{axis} \quad(y=0)$

1. Perpendicular to $x$-axis Cross Sections Squares
2. Perpendicular to $x$-axis Cross Sections Equilateral Triangles

Base: $y=\frac{1}{2} x^{2}$
$y=-x^{2}+6$

5. Perpendicular to $x$-axis

Cross Sections Squares
7. Perpendicular to $x$-axis

Cross Sections Equilateral Triangles
Partner
$\qquad$
$\qquad$ Day 3 \& 4 App Integration

2. Perpendicular to $y$-axis

Cross Sections Isosceles Right Triangles with leg on base.
4. Perpendicular to $y$-axis

Cross Sections Semi-Circles
6. Perpendicular to $y$-axis

Cross Sections Isosceles Right Triangles with leg on base.
8. Perpendicular to $y$-axis

Cross Sections Semi-Circles
$\qquad$
$\qquad$ Day 3 \& 4 App Integration

Base: $y=4 \sin \left(\frac{1}{2} x\right)$ $y=x^{2}-6 x$
9. Perpendicular to $x$-axis Cross Sections Squares

11. Perpendicular to $x$-axis

Cross Sections Equilateral Triangles
10. Perpendicular to $x$-axis Cross Sections Isosceles Right Triangles with leg on base
12. Perpendicular to $x$-axis Cross Sections Semi-Circles
13. Let $f$ and $g$ be the functions defined by $f(x)=1+x+e^{x^{2}-2 x}$ and $g(x)=x^{4}-6.5 x^{2}+6 x+2$ let $R$ and $S$ be the two regions enclosed by the graphs of $f$ and $g$
 shown in the figure to the right.

$$
\text { 14. Let } f(x)=2 x^{2}-6 x+4 \text { and }
$$

$$
g(x)=4 \cos \left(\frac{1}{4} \pi x\right) . \text { Let } R
$$ be the region bounded by the graphs of $f$ and $g$, as shown in the figure.



Region $S$ is the base of a solid whose cross sections perpendicular to the $x$ axis are squares. Find the volume of the solid.

The region $R$ is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.
$\qquad$
Partner Day 3 \& 4 App Integration
15. Let $R$ be the region in the first quadrant bounded by the x-axis and the graphs of
$y=\ln x$ and $y=5-x$, as shown in the figure to the right.
16. Let $R$ be the region in the first quadrant
bounded by the graph of $y=2 \sqrt{x}$, the horizontal line $y=6$, and the $y$ axis, as shown in the figure to the right.

17. Let $\mathfrak{R}$ be the region in the first quadrant enclosed by the graphs of $y=2 x$ and $y=x^{2}$, as shown in the figure to the right.

18. The base of a solid is the region enclosed by $y=\sin x$ and the $x$-axis on the interval $[0, \pi]$. Cross sections perpendicular to the x-axis are semicircles with diameter in the plane of the base. Write an integral that represents the volume of the solid.
(A) $\frac{\pi}{8} \int_{0}^{\pi}(\sin x)^{2} d x$
(B) $\frac{\pi}{8} \int_{0}^{1}(\sin x)^{2} d x$
(C) $\frac{\pi}{4} \int_{0}^{\pi} \sin x d x$
(D) $\frac{\pi}{8} \int_{0}^{\pi}(\sin x)^{2} d x$
(E) $\frac{\pi}{2} \int_{0}^{\pi}(\sin x)^{2} d x$
19. The base of a solid is a region in the first quadrant bounded by the $x$-axis, $y$-axis, and the line $x+2 y=8$, as shown in the figure. If the cross sections of the solid perpendicular to the $x$-axis are semicircles, what is the volume of the solid?
(A) 12.566
(B) 14.661
(C) 16.755

(D) 67.021
(E) 134.041

Region $R$ is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression involving one or more integrals that gives the volume of the solid.

Region $R$ is the base of a solid. For each $y$, where $0 \leq y \leq 6$, the cross section of the solid taken perpendicular to the $y$-axis is a rectangle whose height is 3 times the length of its base in region $R$. Write, but do not evaluate, an integral expression that gives the volume of the solid.

Find the region $\mathfrak{R}$ is the base of a solid, at each $x$ the cross section perpendicular to the $x$-axis has area $A(x)=\sin \left(\frac{\pi}{2} x\right)$. Find the volume of the solid.
20. The base of a solid is the region in the first quadrant bounded by the $y$-axis, the graph of $y=\tan ^{-1} x$, the horizontal line $y=3$, and the vertical line $x=1$. For this solid, each cross section perpendicular to the $x$-axis is a square. What is the volume of the solid?
(A) 2.561
(B) 6.612
(C) 8.046
(D) 8.755
(E) 20.773

