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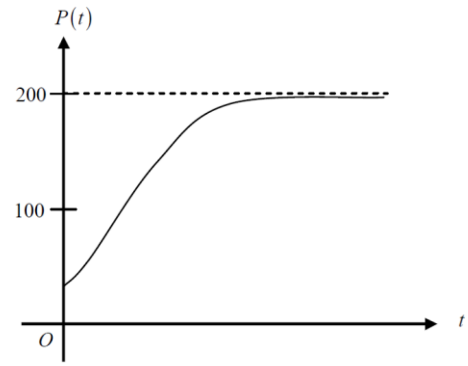
Logistic Equation & Euler's Method Day 5 Date____________________Period________

1. Given that $y(1) = -3$ and $\frac{dy}{dx} = 2x + y$, what is the approximation for $y(2)$ if Euler's method is used with a step size of .05, starting at x=1?

(A) -5
(B) -4.25
(C) -4
(D) -3.75
(E) -3.5

2. Which of the following differential equations for a population P could model the logistic growth shown in the figure?

(A) $\frac{dP}{dt} = 0.2P - 0.001P^2$
(B) $\frac{dP}{dt} = 0.1P - 0.001P^2$
(C) $\frac{dP}{dt} = 0.2P^2 - 0.001P$
(D) $\frac{dP}{dt} = 0.1P^2 - 0.001P$
(E) $\frac{dP}{dt} = 0.1P^2 + 0.001P$



3. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x + y$ with the initial condition $f(1) = 2$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x = 1$ with a step size of 0.5?

(A) 3
(B) 5
(C) 6
(D) 10
(E) 12

4. The number of moose in a national park is modeled by the function M that satisfies the logistic differential equation $\frac{dM}{dt} = 0.6M(1 - \frac{M}{200})$, where t is the time in years and $M(0) = 50$. What is $\lim_{t \to \infty} M(t)$?

(A) 50
(B) 200
(C) 500
(D) 1000
(E) 2000

5. The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$, where the initial population $P(0) = 3{,}000$ and t is the time in years. What is $\lim_{t \to \infty} P(t)$?

(A) $2{,}500$
(B) $3{,}000$
(C) $4{,}200$
(D) $5{,}000$
(E) $10{,}000$

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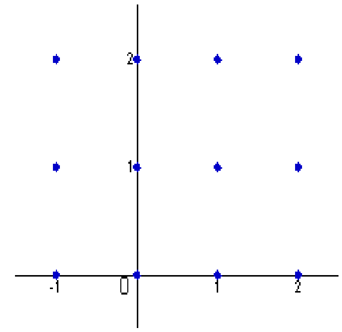
No Calculator Problem

6. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

a.) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and sketch the solution curve that passes through the point $(0,1)$



b.) The solution curve that passes through the point $(0,1)$ has a local minimum at $x = \ln\left(\frac{3}{2}\right)$.

What is the y – coordinate of this local minimum?

c.) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with two steps of equal size to approximate $f(-0.4)$. Show the work that leads to your answer.

d.) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine whether the approximation found in part (c) is less than or greater than $f(-0.4)$. Explain your reasoning.

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7. A population is modeled by a function P that satisfies the logistic differential equation

$$\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{12}\right).$$

a.) If $P(0) = 3$, what is $\lim_{t\to\infty} P(t)$?

If $P(0) = 20$, what is $\lim_{t\to\infty} P(t)$?

---

b.) If $P(0) = 3$, for what value of P is the population growing fastest?

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c.) A differential population is modeled by a function Y that satisfies the separable differential equation

$$\frac{dY}{dt} = \frac{Y}{5}\left(1 - \frac{t}{12}\right).$$

Find $Y(t)$ if $Y(0) = 3$.

---

d.) For the function Y found in part (c), what is $\lim_{t\to\infty} Y(t)$?

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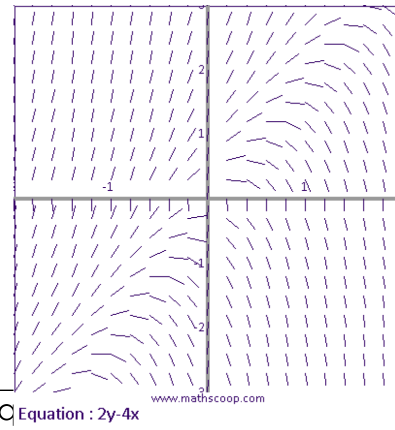
8. Consider the differential equation $\frac{dy}{dx} = 2y - 4x$

a.) The slope field for the given differential equation is provided. Sketch the solution curve that passes through the point (0,1) and sketch the solution curve that passes through the point (0,-1).



c.) Let f be the function that satisfies the given differential eq... tion $f(0) = 1$. Use Euler's method, starting at $x = 0$ with a step size of 0.1 to approximate $f(.2)$. Show the work that leads to your answer.

d.) Find the value of b for which $y = 2x + b$ is a solution to the given differential equation. Justify your answer.

e.) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. Does the graph of g have a local extremum at the point $(0,0)$? If so, is the point a local maximum or a local minimum? Justify your answer.

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9. Let f be the function satisfying $f'(x) = -3xf(x)$, for all real numbers x, with $f(1) = 4$ and $\lim_{x \to \infty} f(x) = 0$
.

a.) Evaluate $\int_1^{\infty} -3xf(x)dx$. Show the work that leads to your answer.

---

b.) Use Euler's method, starting at $x = 1$ with a step size of 0.5, to approximate $f(2)$.

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c.) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = -3xy$ with the initial condition $f(1) = 4$.

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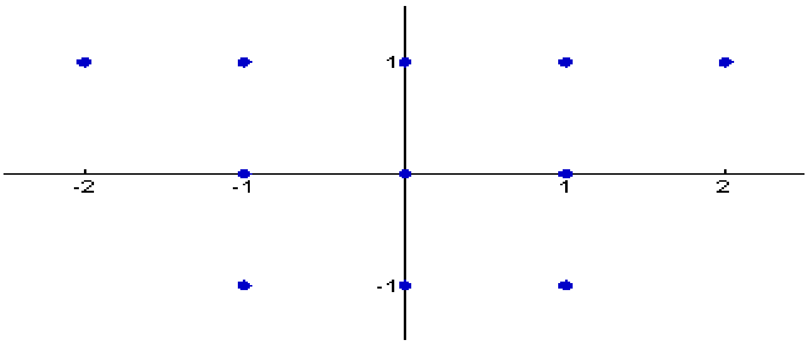
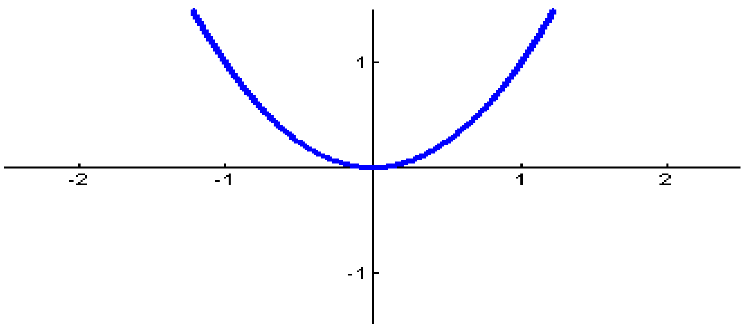
10. Consider the differential equation given by $\frac{dy}{dx} = x(y-1)^2$

a.) On the axis provided, sketch a slope field for the given differential equation at the eleven points indicated.



b.) Use the slope field for the given differential equation to explain why a solution could not have the graph shown below.



c.) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -1$.

d.) Find the range of the solution found in part (c).

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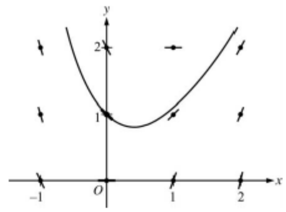
Answers:

1. D
2. A
3. C
4. B
5. E

6.a. 

b. $y = 2\ln\left(\frac{3}{2}\right)$

c. $f(-.4) \approx 1.52$

d. $\left.\frac{d^2y}{dx^2}\right|_{(0,1)} > 0$

$\therefore f(x)$ *is concave up*

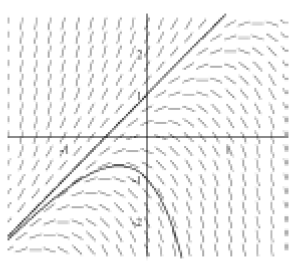
so Underapproximation

7a. 12

b. 6

c. $Y = 3e^{\frac{1}{5}t - \frac{1}{120}t^2}$

d. 0

8a. 

b. $f(.2) \approx 1.4$

c. $b = 1$

d. $g(0) = 0$ *is a* max

b.c. $g'(0) = 0$ &

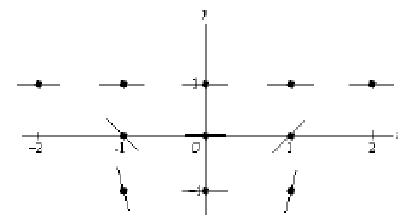
$g''(0) < 0$

9.a. -4

b. $f(2) \approx 2.5$

c. $y = 4e^{-\frac{3}{2}x^2 + \frac{3}{2}}$

d.

10a. 

b. Slope field $\frac{dy}{dx} = 0$ at $y = 1$ and the graph does not.

c. $y = -\frac{2}{x^2+1} + 1$

d. $[-1, 1]$