AP Calculus BC | Name______________________________ Pd.____

Supplement: Logistic Equations & Euler's Method | Additional Differential Equations Day 4

1. Determine the growth rate, the carrying capacity, find y(t), and y(10). Given $\frac{dy}{dt}=3y\left(1-\frac{y}{5}\right)$ and $y(0)=2$.

$C=\frac{y_0}{y_0-5}$

$C=\frac{2}{2-5}=\frac{-2}{3}$

growth Rate (K) = 3

carrying capacity (A) = 5

$y=\frac{5}{1-\frac{e^{-3t}}{-2/3}}=\frac{(5)2}{(1+\frac{3}{2}e^{-3t})2}=\frac{10}{2+3e^{-3t}}$

$y(10)=\frac{10}{2+3e^{-3(10)}}=$

2. Determine the growth rate, the carrying capacity, find y(t), and y(5). Given $\frac{dy}{dt}=2y(3-y)$ and $y(0)=10$.

$y=\frac{3}{1-\frac{e^{-6t}}{10/7}}$

$\frac{dy}{dt}=2y(3)\left[1-\frac{y}{3}\right]$

$y=\left(\frac{3}{1-\frac{7}{10}e^{-6t}}\right)\frac{10}{10}$

$\frac{dy}{dt}=6y\left[1-\frac{y}{3}\right]$

$y=\frac{30}{10-7e^{-6t}}$

growth Rate (K) = 6

carrying capacity (A) = 3

$C=\frac{10}{10-3}=\frac{10}{7}$

3. $\dot{y}=.5y(1-.5y)$, $y(0)=4$ Determine $\lim_{t\to\infty}y(t)$.

$\dot{y}=.5y\left[1-\frac{1}{2}y\right]$

$\dot{y}=.5y\left[1-\frac{y}{2}\right]$

carrying capacity = 2

$\lim_{t\to\infty}y(t)=2$

4. Find $\lim_{t\to\infty}y(t)$

If A.) A=15 B.) A=5 C.) A=10

$\lim_{t\to\infty}y(t)=15$ $\lim_{t\to\infty}y(t)=5$ $\lim_{t\to\infty}y(t)=10$

limit = carrying capacity

5. A population of squirre3ls lives in a forest with a carrying capacity of 2,000. Assume logistic growth with growth constant $k=.6yr^{-1}$.

A.) Find a formula for the squirrel population P(t), assuming an initial population of 500 squirrels.

$C=\frac{500}{500-2000}=-\frac{1}{3}$ $y=\frac{2000}{1-\frac{e^{-.6t}}{-1/3}}=\frac{2000}{1+3e^{-.6t}}$

B.) How long will it take for the population to double? double then $y=1000$

$1{,}000=\frac{2{,}000}{1+3e^{-.6t}}$ $1+3e^{-.6t}=2$

$3e^{-.6t}=1$ $\cancel{\ln}e^{-.6t}=\ln\frac{1}{3}$ $-.6t=\ln(1/3)$ $t=\frac{\ln(1/3)}{-.6}=1.831$ years

6. The population P(t) of mosquito larvae growing in a tree hole increases according to the logistic equation with growth constant $k=.3days^{-1}$ and carrying capacity A=500.

A.) Find a formula for the larvae population P(t), assuming initial population $P_0=50$ larvae.

$C=\frac{50}{50-500}=-\frac{1}{9}$ $y=\frac{500}{1-\frac{e^{-.3t}}{-1/9}}=\frac{500}{1+9e^{-.3t}}$

B.) After how many days will the larvae population reach 200?

$200=\frac{500}{1+9e^{-.3t}}$ $1+9e^{-.3t}=\frac{500}{200}$ $1+9e^{-.3t}=5/2$

$\left(\frac{1}{9}\right)9e^{-.3t}=3/2\left(\frac{1}{9}\right)$

$\cancel{\ln}e^{-.3t}=\ln\frac{1}{6}$

$-.3t=\ln(1/6)$

$t=\frac{\ln(1/6)}{-.3}$

$t=5.9725$ days

7. Sunset lake is stocked with 2,000 rainbow trout and after 1 year of population has grown to 4500. Assuming logistic growth with a carrying capacity of 20,000, find the growth constant k and determine when the population will reach 10,000. $(1, 4500)$ $y_0 = 2{,}000$ $A = 20{,}000$

$C = \frac{2{,}000}{2{,}000 - 20{,}000} = \frac{-2{,}000}{18{,}000} = \frac{-1}{9}$

$y = \frac{20{,}000}{1 - \frac{e^{-kt}}{-1/9}} = \frac{20{,}000}{1 + 9e^{-kt}}$

$4500 = \frac{20{,}000}{1 + 9e^{-k(1)}}$

$1 + 9e^{-k} = \frac{20{,}000}{4500}$

$\cancel{1} + 9e^{-k} = \frac{40}{9} - \frac{9}{9}$

$\frac{1}{9}(9e^{-k} = \frac{31}{9})$

$e^{-k} = \frac{31}{81}$

$-k = \ln(\frac{31}{81})$

$\boxed{k = .9605}$

$10{,}000 = \frac{20{,}000}{1 + 9e^{-.9605t}}$

$\cancel{1} + 9e^{-.9605t} = 2 - 1$

$e^{-.9605t} = 1/9$

$t = \frac{\ln(1/9)}{-.9605} = \boxed{2.2879 \text{ years}}$

8. A rumor spreads through a small town. Let y(t) be the fraction of the population that has heard the rumor at time t and assume that the rate at which the rumor spreads is proportional to the product of the fraction y of the population that has heard the rumor and the fraction 1-y that has not heard the rumor.

A.) Write down the differential equation satisfied by y in terms of a proportionality factor k.

$\dot{y} = ky(1 - y)$

B.) Find k, assuming 10% of the population knows the rumor at t=0 and 40% knows it 2 days later. $(2, .4)$

carrying capacity = 100% = 1

$y_0 = 10\% = 1/10$

$C = \frac{1/10}{1/10 - 1} = \frac{1/10}{-9/10}\left(\frac{-10}{9}\right) = \frac{-1}{9}$

$y = \frac{1}{1 - \frac{e^{-kt}}{-1/9}}$

$y = \frac{1}{1 + 9e^{-kt}}$

$.4 = \frac{1}{1 + 9e^{-2k}}$

$1 + 9e^{-2k} = \frac{1}{.4}$

$\cancel{1} + 9e^{-2k} = 10/4 - 4/4$

$(1/9)9e^{-2k} = 5/4(1/9)$

$e^{-2k} = 5/36$

$k = \frac{\ln(5/36)}{-2} = .8959$

$\boxed{k = .8959}$

C.) Determine when 75% will know.

$.75 = \frac{1}{1 + 9e^{-.8959t}}$

$1 + 9e^{-.8959t} = \frac{1}{.75}$

$1 + 9e^{-.8959t} = 1/3/4$

$\cancel{1} + 9e^{-.8959t} = 4/3 - 3/3$

$(\frac{1}{9})9e^{-.8959t} = 1/3(\frac{1}{9})$

$e^{-.8959t} = 1/27$

$t = \frac{\ln(1/27)}{-.8959}$

$\boxed{t = 3.6788 \text{ days}}$

9. A rumor spreads though a school with 1,000 students. At 8am, 80 students have heard the rumor and by noon, half the school has heard it. Determine when 90% of the students will have heard it. 8 AM $t = 0$ NOON $t = 4$ $(4, 500)$

$C = \frac{80}{80 - 1{,}000} = \frac{80}{-920} = -\frac{2}{23}$

$y = \frac{(1{,}000)2}{\left(1 + \frac{23e^{-kt}}{2}\right)2} = \frac{2{,}000}{2 + 23e^{-kt}}$

$500 = \frac{2{,}000}{2 + 23e^{-k(4)}}$

$\cancel{2} + 23e^{-4k} = 4 - 2$

$23e^{-4k} = 2$

$e^{-4k} = 2/23$

$k = \frac{\ln(2/23)}{-4}$

$\boxed{k = .6106}$

90% of 1,000 = 900

$900 = \frac{2000}{2 + 23e^{-.6106t}}$

$2 + 23e^{-.6106t} = \frac{2000}{900}$

$\cancel{2} + 23e^{-.6106t} = \frac{20}{9} - \frac{18}{9}$

$(\frac{1}{23})23e^{-.6106t} = \frac{2}{9}(\frac{1}{23})$

$t = \frac{\ln(\frac{2}{207})}{-.6106} = \boxed{7.5984 \text{ hours}}$