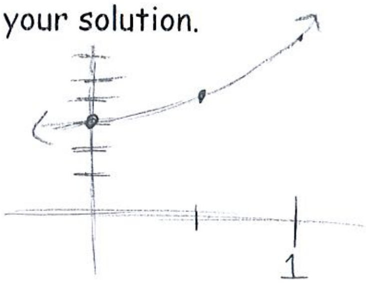
BC Calculus — Name________________________ Pd.___

Supplement Euler's Method — Additional Differential Equations Day 2

1. A) Given the differential equation $\frac{dy}{dx} = x + 2$ and $y(0) = 3$. Find an approximation for y(1) by using Euler's method with two equal steps. Sketch your solution.

0	$(x+2)(.5)$	3
.5	$[0+2](.5)=1$	4
1	$[.5+2](.5)=1.25$	5.25



B.) Solve the differential equation $\frac{dy}{dx} = x + 2$ with the initial condition $y(0) = 3$, and use your solution to find y(1).

x y

$dy = (x+2)\,dx$

$y = \frac{1}{2}x^2 + 2x + C$

$3 = \frac{1}{2}(0)^2 + 2(0) + C$

$C = 3$

$y = \frac{1}{2}x^2 + 2x + 3$

$y(1) = \frac{1}{2}(1)^2 + 2(1) + 3 = 5.5$

C.) The error in using Euler's method is the difference between the approximate value and the exact value. What was the error in your answer? How could you produce a smaller error using Euler's method?

ERROR = .25

$\left|\frac{\text{Actual} - \text{Approx}}{\text{Actual}}\right| \cdot 100 = \left|\frac{5.5 - 5.25}{5.5}\right| 100 = 4.545\%$

The smaller Δx is then the smaller the error.

2. Suppose a continuous function f and its derivative f' have values that are given in the following table. Given that f(2)=5, use Euler's method with two steps of size $\Delta x = 0.5$ to approximate the value of f(3).

x	2.0	2.5	3.5
f'(x)	0.4	0.6	0.8
f(x)	5		

2	$f'(x) \cdot \Delta x$	5
2.5	$f'(2)[.5]$ $(.4)(.5) = .2$	5.2
3	$f'(2.5)[.5]$ $(.6)[.5] = .3$	5.5

$\boxed{y(3) \simeq 5.5}$

3. Given the differential equation $\frac{dy}{dx} = \frac{1}{x+2}$ and $y(0) = 1$. Find an approximation of $y(1)$ using Euler's method with two steps and step size $\Delta x = 0.5$.

0	$\left(\frac{1}{x+2}\right)(.5)$	1
.5	$\left(\frac{1}{0+2}\right)(.5) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = .25$	1.25
1	$\left(\frac{1}{.5+2}\right)(.5) = .2$	1.45

$\boxed{y(1) \approx 1.45}$

4. Given the differential equation $\frac{dy}{dx} = x + y$ and $y(1) = 3$. Find an approximation of $y(2)$ using Euler's method with two equal steps. $\Delta x = .5$

1	$[x+y](.5)$	3
1.5	$[1+3](.5) = 2$	5
2	$[1.5+5](.5) = 3.25$	8.25

$\boxed{y(2) \approx 8.25}$

5. The curve passing through (2,0) satisfies the differential equation $\frac{dy}{dx} = 4x + y$. Find an approximation to $y(3)$ using Euler's method with two equal steps.

2	$[4x+y](.5)$	0
2.5	$[4(2)+0](.5) = 4$	4
3	$[4(2.5)+4](.5) = 7$	11

$\boxed{y(3) \approx 11}$

6. Assume that f and f' have the values given in the table. Use Euler's method with two equal steps to approximate the value of f(4.4).

x	4	4.2	4.4
f'(x)	-0.5	-0.3	-0.1
f(x)	2		

$\boxed{f(4.4) \approx 1.84}$

4	$f'(x)[.2]$	2
4.2	$f'(4)[.2]$ $(-.5)(.2) = -.1$	1.9
4.4	$f'(4.2)[.2]$ $(-.3)(.2) = -.06$	1.84

7. Let y=f(x) be the solution to the differential equation $\frac{dy}{dx} = \arcsin(xy)$ with the initial condition $f(0) = 2$. What is the approximation for f(1) if Euler's method is used, starting at x=0 with step size of 0.5?

A.)	2	B.)	$2+\frac{\pi}{6}$	C.)	$2+\frac{\pi}{4}$	D.)	$2+\frac{\pi}{2}$	E.)	3

0	$[\arcsin(xy)][.5]$	2
.5	$\arcsin(0(.5))(.5)$ $\arcsin(0)(.5)$ $0(.5) = 0$	2
1	$\arcsin\left(\frac{1}{2}(2)\right)[.5]$ $\arcsin(1)[.5]$ $\frac{\pi}{2}\left[\frac{1}{2}\right] = \frac{\pi}{4}$	$2+\frac{\pi}{4}$

Answer = C
$f(1) \approx 2 + \frac{\pi}{4}$