The Derivative of the Inverse Name Derivatives (2) Day 5
Classwork
Assume that $\mathrm{f}(\mathrm{x})$ is differentiable and one-to-one with inverse $f^{-1}(x)$. If b belongs to the domain of $f^{-1}(x)$ and $f^{\prime}\left(f^{-1}(x)\right) \neq 0$, then $\left(f^{-1}\right)^{\prime}(b)$ exists and

$$
\left(f^{-1}\right)^{\prime}(\mathrm{AT})=\frac{1}{f^{\prime}\left(f^{-1}(\mathrm{AT})\right)} \bullet \frac{d}{d x}[A T]
$$

1. If $f(4)=5$ and $f^{\prime}(4)=\frac{2}{3}$, find $\left(f^{-1}\right)^{\prime}(5)$.

2-7: Use the table to the right.

| x | $\mathrm{f}(\mathrm{x})$ | $\mathrm{f}^{\prime}(\mathrm{x})$ |
| :---: | :---: | :---: |
| 2 | 3 | 4 |
| 3 | $\frac{35}{4}$ | $\frac{31}{4}$ |
| 4 | 19 | 13 |


| 2. Find $f^{-1}(3)$ | 3. Find $\left(f^{-1}\right)^{\prime}(3)$ | 4. Find $f^{-1}(x)$ when $x=19$. |
| :--- | :--- | :--- |
| 5. Find $\left(f^{-1}\right)^{\prime}(19)$. | 6. Find $f^{-1}\left(\frac{35}{4}\right)$. | 7. Find $\left(f^{-1}\right)^{\prime}\left(\frac{35}{4}\right)$. |

8. Suppose that $P=(2,4)$ lies on the graph of $f(x)$ and that the slope of the tangent line through P is $\mathrm{m}=3$. Assuming that $f^{-1}(\mathrm{x})$ exists, what is the slope of the tangent line to the graph of $f^{-1}(x)$ at the point $Q=(4,2)$ ?
9. If $f(x)=2 x+7$, find $\left(f^{-1}\right)^{\prime}(3)$.
10. If $\mathrm{f}(\mathrm{x})=x^{2}-1$ has restricted domain $[0, \infty)$, find $\left(f^{-1}\right)^{\prime}(15)$.
Answers: 1) $\frac{3}{2}$
2) 2 3) $\frac{1}{4}$ 4) $4 \quad$ 5) $\frac{1}{13}$
3) 3 7) $\frac{4}{31} 8$ ) $\frac{1}{3}$
4) $1 / 2$ 10) $\frac{1}{8}$

For problems 1-2, find $\frac{d}{d x}\left(f^{-1}(x)\right)$ at $\mathrm{x}=\mathrm{a}$
11. $f(x)=4 x^{3}-2 x$ where $\mathrm{a}=-2$
12. $f(x)=\frac{1}{1+x}$ where $\mathrm{a}=\frac{1}{4}$

Derivatives of Inverse Trig. Functions
For problems 3-6, find $f^{\prime}(x)$.

| 13. $f(x)=\sin ^{-1}\left(x^{2}\right)$ | 14. $f(x)=\tan ^{-1}(1-x)$ |
| :--- | :--- |
| 15. $f(x)=\sin ^{-1}\left(e^{x}\right)$ | 16. $f(x)=\tan ^{-1}\left(\frac{1}{x}\right)$ |

Answers: 11) $\frac{1}{10}$
12) -16 13) $\frac{2 x}{\sqrt{1-x^{4}}}$
14) $\frac{-1}{2-2 x+x^{2}}$
15) $\frac{e^{x}}{\sqrt{1-e^{2 x}}}$
16) $\frac{-1}{x^{2}+1}$

