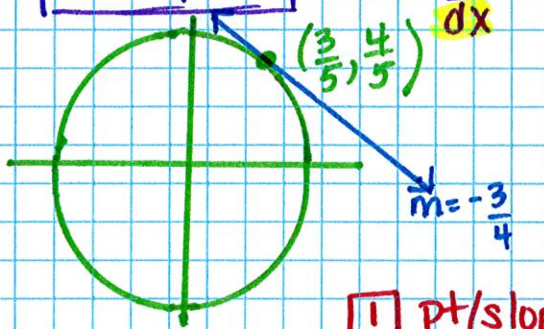
Notes: Implicit Differentiation

- Used when taking a derivative and variables do not match.

Not Implicit — Variables match	Implicit — Variables do not match
$\frac{d}{dx}[3x^2] = 6x$	$\frac{d}{dx}[3y^2] = 6y \cdot \frac{dy}{dx}$
$\frac{d}{dy}[\cos(2y)] = -2\sin(2y)$	$\frac{d}{dy}[\cos(2x)] = -2\sin(2x)\frac{dx}{dy}$
$\frac{d}{dq}[e^q] = e^q$	$\frac{d}{dt}[e^x] = e^x \frac{dx}{dt}$

$\boxed{\text{Example 1}}$ Find $\frac{dy}{dx}$ at $\left(\frac{3}{5}, \frac{4}{5}\right)$ for $\frac{d}{dx}[x^2 + y^2 = 1]$. Find the tangent line and normal line.



$$\frac{d}{dx}[x^2] + \frac{d}{dx}[y^2] = \frac{d}{dx}[1]$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y} = \frac{-\frac{3}{\cancel{5}}}{\frac{4}{5}} \cdot \frac{\cancel{5}}{4} = -\frac{3}{4}$$

$\boxed{1}$ pt/slope $y - y_1 = m(x - x_1)$

$$y - \frac{4}{5} = -\frac{3}{4}\left(x - \frac{3}{5}\right)$$

$\boxed{2.}$ slope/intercept

$$\frac{4}{5} = \left(-\frac{3}{4}\right)\left(\frac{3}{5}\right) + b$$

$$\frac{4 \cdot 4}{4 \cdot 5} = -\frac{9}{20} + b$$

$$+\frac{9}{20} \qquad +\frac{9}{20}$$

$$b = \frac{25}{20} = \frac{5}{4}$$

$$y = -\frac{3}{4}x + \frac{5}{4}$$

$\boxed{3.}$ Standard Form

$$4\left(y = -\frac{3}{4}x + \frac{5}{4}\right) \qquad 4y = -3x + 5 \qquad 3x + 4y = 5$$

Normal Line $\left(\frac{3}{5}, \frac{4}{5}\right)$ $m = \frac{4}{3}$ $\quad y - \frac{4}{5} = \frac{4}{3}\left(x - \frac{3}{5}\right)$

Example 2 Find the derivative with Respect to x.

$$y^4 + xy = x^3 - x + 2$$

Product Rule

$$\frac{d}{dx}[y^4] + \left[x\frac{d}{dx}(y) + y\frac{d}{dx}(x)\right] = \frac{d}{dx}[x^3] - \frac{d}{dx}[x] + \frac{d}{dx}[2]$$

$$4y^3\frac{dy}{dx} + x(1)\frac{dy}{dx} + y(1) = 3x^2 - 1$$

$$4y^3\frac{dy}{dx} + x\frac{dy}{dx} = 3x^2 - 1 - y$$

$$\frac{dy}{dx}\frac{\cancel{(4y^3+x)}}{\cancel{4y^3+x}} = \frac{3x^2-1-y}{4y^3+x} = \frac{3x^2-1-y}{4y^3+x}$$

Solve for $\frac{dy}{dx}$

1. move all terms with dy/dx on left side of equation and all others to Right
2. Factor out dy/dx
3. Divide

Example 3 Find $\frac{dy}{dt}$, where $\cos(ty) = \frac{t^2}{y}$

$$-\sin(ty)\cdot\left[t\frac{d}{dt}(y) + y\frac{d}{dt}(t)\right] = \frac{y\frac{d}{dt}(t^2) - t^2\frac{d}{dt}(y)}{y^2}$$

$$-\sin(ty)\left[t(1)\frac{dy}{dt} + y(1)\right] = \frac{y(2t) - t^2(1)\frac{dy}{dt}}{y^2}$$

$$y^2\left(-t\sin(ty)\frac{dy}{dt} - y\sin(ty)\right) = \left(\frac{2ty - t^2\frac{dy}{dt}}{\cancel{y^2}}\right)\cancel{y^2}$$

$$-ty^2\sin(ty)\frac{dy}{dt} - y^3\sin(ty) = 2ty - t^2\frac{dy}{dt}$$

$$-ty^2\sin(ty)\frac{dy}{dt} + t^2\frac{dy}{dt} = 2ty + y^3\sin(ty)$$

$$\frac{dy}{dt}\frac{\cancel{(-ty^2\sin(ty)+t^2)}}{\cancel{-ty^2\sin(ty)+t^2}} = \frac{2ty + y^3\sin(ty)}{-ty^2\sin(ty)+t^2}$$

$$\frac{dy}{dt} = \frac{2ty + y^3\sin(ty)}{-ty^2\sin(ty)+t^2}$$