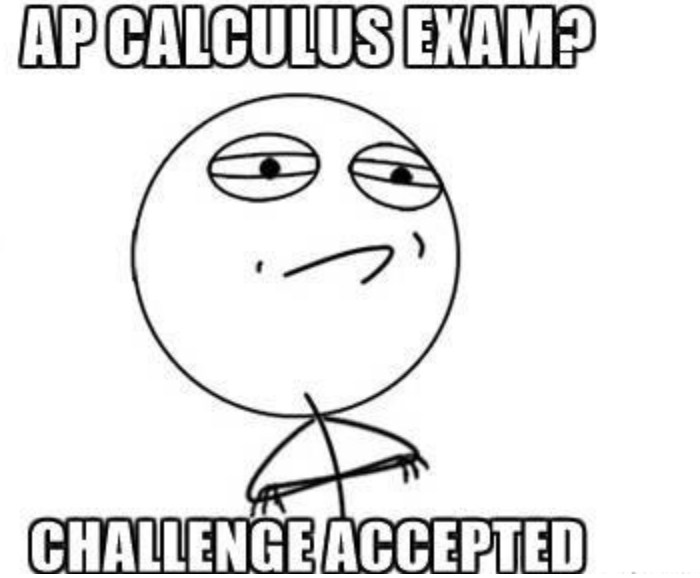
AP Calculus BC



Exam Review

Here's what to expect for the next 5 weeks:

1. Every assignment is graded, and **NO LATE PAPERS WILL BE ACCEPTED**. If you are going to miss class, please have someone bring your paper to me when it is due.

2. All graded assignments are determined on an AP scale as follows:

Percent	AP Grade	Class Grade (out of 100%)
84-100	5	100
68-83	4	90
45-67	3	80
29-44	2	70
1-28	1	60
0	0	0

The mock exam will follow a slightly different scale.

3. To help with the grading process, please turn in homework assignments at the beginning of class and classwork assignments before you leave class. Remember, no late papers can be accepted!

4. Bring your notebook and notecards to class everyday – you may leave your textbooks at home, if you like.

ABOUT THE EXAM

Part I: Multiple Choice (1 hour and 45 minutes total, 45 questions)

Section A (60 minutes): 30 multiple choice questions. Calculators are not allowed on this portion of the exam.

Section B (45 minutes): 15 multiple choice questions. Any TI calculator except the TI-92 is allowed on this portion of the exam. You may use 2 approved calculators on the exam. Only about 1/3 of questions will need a calculator, however; the remainder are generally theory or longer "thinking" questions.

Part II: Free Response (90 minutes, 6 multi-part questions)

Section A (30 minutes): 2 problems for which calculators may be used.

Section B (60 minutes): 4 problems – no calculators allowed. In addition, you may continue to work on the first 2 free-response problems, without calculator, time permitting.

General Tips for the Exam

Show all work.
Always show how you set up the problem, even if you use the calculator to find the derivative or integral. An answer without an integral will not get full credit, even if it is correct. Graphs or sign charts used as justification MUST be labeled.

Do not round partial answers.
Never round within the problem; round answers to 3 decimal places (AP) or 3 sigfigs (IB) only at the last step of the problem.

Do not let the points at the beginning keep you from getting the points at the end.
If you can do part (c) without doing (a) and (b), do it. If you need to import an answer from part (a), make a credible attempt at part (a) so that you can import the (possibly wrong) answer and get your part (c) points.

If you use your calculator to solve an equation, write the equation first.
An answer without an equation might not get full credit, even if it is correct.

Do not waste time erasing bad solutions.
If you change your mind, simply cross out the bad solution after you have written the good one. *Crossed-out work will not be graded*. If you have no better solution, leave the old one there. It might be worth a point or two.

Do not use your calculator for anything except:
(a) graph functions, (b) compute numerical derivatives, (c) compute definite integrals, and (d) solve equations. In particular, do not use it to determine max/min points, concavity, inflection points, increasing/decreasing, domain, and range. (You can explore all these with your calculator, but your solution must stand alone.)

Be sure you have answered the problem.
For example, if it asks for the maximum value of a function, do not stop after finding the x at which the maximum value occurs.

Always include units.
Be sure to express your answer in correct units if units are given.

If you can eliminate some incorrect answers in the multiple-choice section, it is advantageous to guess.
Otherwise it is not. Wrong answers can often be eliminated by estimation, or by thinking graphically. Also, never select more than 4 of the same MC responses in a row.

If they ask you to justify your answer, think about what needs justification.
They are asking you to say more. If you can figure out why, your chances are better of telling them what they want to hear. For example, if they ask you to justify a point of inflection, they are looking to see if you realize that a sign change of the second derivative must occur.

Know the Lingo
Accumulator function is an integral with a variable in the limits. *Difference Quotient* is the slope formula. *Symmetric Derivative* is the limit of the difference quotient taken at equal "distances" from the point in question. *Increasing at a decreasing rate* implies a climbing function that is concave down; *decreasing at a decreasing rate* implies a falling function that's concave up, etc.; a picture will help you with these concepts.

Tables #1
Non Calculator

Name___________________
Date____________Period__

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

a.) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

b.) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

c.) Use a midpoint sum with three subintervals of equal length indicated by the date in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)dt$ in the context of the problem.

d.) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

Tables #2
Non-Calculator

Name__________________
Date___________Period___

x	2	3	5	8	13
$f(x)$	1	4	-2	3	6

Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for select points in the closed interval $2 \le x \le 13$.

a.) Estimate $f'(4)$. Show the work that leads to your answer.

b.) Evaluate $\int_{2}^{13} \left(3 - f'(x)\right)\,dx$. Show the work that leads to your answer.

c.) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_{2}^{13} f(x)\,dx$. Show the work that leads to your answer.

Tables #3
Non-Calculator

Name____________________
Date_______________Pd.___

Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where t is measured in minutes. Selected values for $v_A(t)$ are given in the table.

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	-120	-150

a.) Find the average acceleration of train A over the interval $2 \le t \le 8$.

b.) Do the data in the table support the conclusion that train A's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.

c.) At time $t = 2$, train A's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A, in meters from the Origin Station, at time $t = 12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t = 12$.

d.) A second train, train B, travels north from the Origin Station. At time t the velocity of train B is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t = 2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train A and train B is changing at time $t = 2$.

Applications #1
Calculator

Name___________________
Date____________Period___

The rate at which people enter an auditorium for a rock concert is modeled by the function *R* given by $R(t) = 1380t^2 - 675t^3$ for $0 \le t \le 2$ hours; *R*(*t*) is measured in people per hour. No one is in the auditorium at time *t*=0, when the doors open. The doors close and the concert begins at time *t*=2.

a.) How many people are in the auditorium when the concert begins?

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b.) Find the time when the rate at which the people enter the auditorium is a maximum. Justify your answer.

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c.) The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function *w* models the total wait time for all the people who enter the auditorium before time *t*. The derivative of *w* is given by $w'(t) = (2-t)R(t)$. Find *w*(2)-*w*(1), the total wait time for those who enter the auditorium after time *t*=1

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d.) On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).

Applications #2
Calculator

Name__________________
Date___________Period___

Mighty Cable Company manufactures cable that sells for $120 per meter. For a cable of fixed length, the cost of producing a portion of the cable varies with its distance from the beginning of the cable. Mighty reports that the cost to produce a portion of a cable that is x meters from the beginning of the cable is $6\sqrt{x}$ dollars per meter. (Note: Profit is defined to be the difference between the amount of money received by the company for selling the cable and the company's cost of producing the cable.)

a.) Find Mighty's profit on the sale of a 25-meter cable.

b.) Using correct units, explain the meaning of $\int_{25}^{30} 6\sqrt{x}dx$ in the context of this problem.

c.) Write an expression, involving an integral, that represents Mighty's profit on the sale of a cable that is k meters long.

d.) Find the maximum profit that Mighty could earn on the sale of one cable. Justify your answer.

Applications #3 Name____________________

Calculator Date_______________Pd.___

Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

a.) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$. Indicate units of measure.

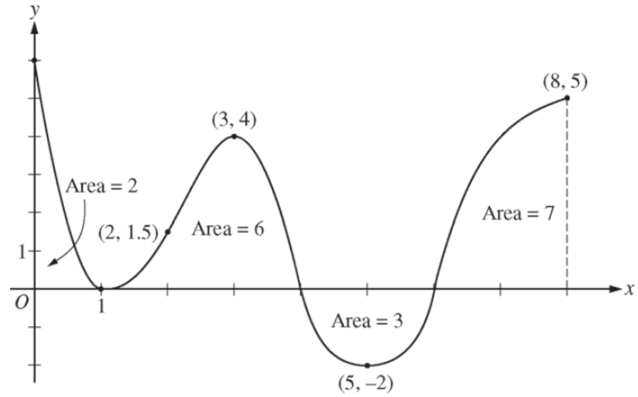
b.) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

c.) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 30$.

d.) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

Graphs #1`
Non-Calculator

Name__________________
Date___________Period__



The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x −axis are labeled in the figure. The function f is defined for all $\mathbb{R}eal$ numbers and satisfies $f(8) = 4$.

a.) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

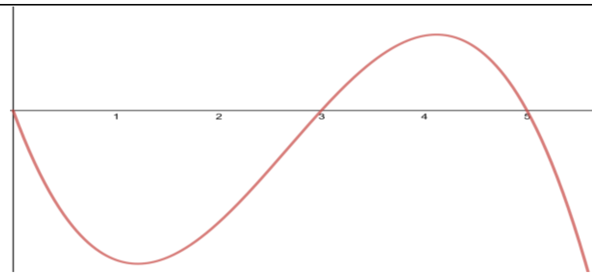
b.) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

c.) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

d.) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

Graphs #2
Non-Calculator

Name___________________
Date____________Period___



A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function *v* whose graph is shown above. The velocity is 0 at *t*=0, *t*=3, and *t*=5, and the graph has horizontal tangents at *t*=1 and *t*=4. The areas of the regions bounded by the *t*-axis and the graph of *v* on the intervals [0, 3], [3, 5], and [5, 6] are 8, 3, and 2, respectively. At time *t*=0, the particle is at x= -2.

a.) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

b.) For how many values of *t*, where $0 \le t \le 6$, is the particle at x= -8? Explain your reasoning.

c.) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

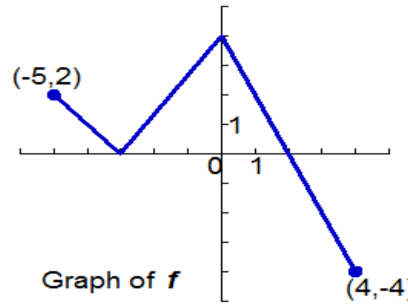
d.) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

Graphs #3
Non-Calculator

Name____________________
Date________________Pd.___

The function f is defined on the closed interval $[-5,4]$. The graph of f consists of three line Segments and is shown in the figure to the right.

Let $g(x)=\int_{-3}^{x} f(t)dt$.



a.) Find $g(3)$.

b.) On what open intervals contained in $-5<x<4$ is the graph of g both increasing and concave down? Give a reason for your answer.

c.) The function h is defined by $h(x)=\dfrac{g(x)}{5x}$. Find $h'(3)$.

d.) The function p is defined by $p(x)=f(x^2-x)$. Find the slope of the line tangent to the graph of p at the point where $x=-1$.

Area/Volume #1
Calculator

Name___________________
Date____________Period___

Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

a.) Find the area of the region enclosed by the graphs of *f* and *g* between $x = \frac{1}{2}$ and x=1.

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b.) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and x=1 is revolved about the line *y*=4.

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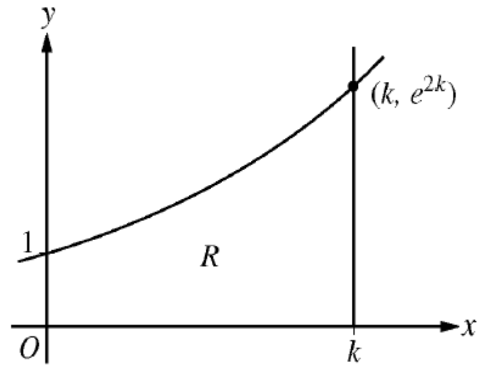
c.) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of *h(*x*)* on the closed interval $\frac{1}{2} \le x \le 1$, and find the absolute maximum value of *h(*x*)* on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.

Name___________________
Date____________Period___

Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of f, the coordinate axes, and the vertical line $x = k$, where $k > 0$. The region R is shown in the figure.



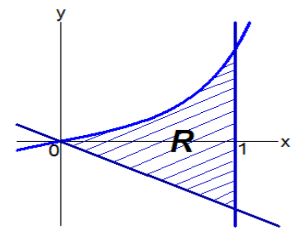
a.) Write, but do not evaluate, an expression involving an integral that gives the perimeter of R in terms of k.

b.) The region R is rotated about the $x-$axis to form a solid. Find the volume, V, of the solid in terms of k.

c.) The volume V, found in part (b), changes as k changes. If $\frac{dk}{dt} = \frac{1}{3}$, determine $\frac{dV}{dt}$ when $k = \frac{1}{2}$

Area/Volume #3 Name____________________
Non-Calculator Date_______________Pd.___

Let R be the shaded region bounded by the graph $y = xe^{x^2}$, the line $y = -2x$, and the vertical line $x = 1$, As shown in the figure to the right.



a.) Find the area of R.

b.) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

c.) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of R.

Differential Equations #1 Name___________________
Non-Calculator Date____________Period___

Consider the differential equation $\frac{dy}{dx} = 1 - y$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(1) = 0$. For this particular solution, $f(x) < 1$ for all values of x.

a.) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(0)$. Shwo the work that leads to your answer.

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b.) Find $\lim_{x \to 1} \frac{f(x)}{x^3 - 1}$. Show the work that leads to your answer.

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c.) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = 1 - y$ with the initial condition $f(1) = 0$.

Differential Equations #2`
Non-Calculator

Name___________________
Date____________Period___

x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

The function f is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f', the derivative of f, are given for selected values of x in the table above.

a.) Write an equation for the line tangent to the graph of f at $x = 1$. Use this line to approximate $f(1.4)$.

b.) Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate $\int_1^{1.4} f'(x)\,dx$. Use the approximation for $\int_1^{1.4} f'(x)\,dx$ to estimate the value of $f(1.4)$. Show the computations that lead to your answer.

c.) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the computations that lead to your answer.

d.) Write the second-degree Taylor polynomial for f about $x = 1$. Use the Taylor polynomial to approximate $f(1.4)$.

Differential Equations #3 Name___________________

Non-Calculator Date____________Period__

Consider the differential equation $\frac{dy}{dx} = y^2(2x+2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$.

a.) Find $\lim_{x\to 0} \frac{f(x)+1}{sinx}$. Show the work that leads to your answer.

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b.) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\frac{1}{2}\right)$.

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c.) Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.

Parametric #1 Name___________________
Calculator Date____________Period___

At time t, a particle moving is the $xy-$plane is at position $(x(t), y(t))$, where $x(t)$ and $y(t)$ are not explicitly given. For $t \geq 0$, $\frac{dx}{dt} = 4t + 1$ and $\frac{dy}{dt} = \sin(t^2)$. At time $t = 0$, $x(0) = 0$ and $y(0) = -4$.

a.) Find the speed of the particle at time $t = 3$, and find the acceleration vector of the particle at time $t = 3$.

b.) Find the slope of the line tangent to the path of the particle at time $t = 3$.

c.) Find the position of the particle at time $t = 3$.

d.) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

Parametric #2
Calculator

Name__________________
Date___________Period___

For $t \geq 0$, a particle is moving along a curve so that its position at time t is $\left(x(t), y(t)\right)$. At time $t = 2$, the particle is at position $(1,5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

a.) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

b.) Find the $x-$coordinate of the particle's position at time $t = 4$.

c.) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

d.) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

Parametric #3 Name____________________
Calculator Date_____________Period___

A particle is moving along a curve so that its position at time t is $\left(x(t), y(t)\right)$, where $x(t) = t^2 - 4t + 8$ and $y(t)$ is not explicitly given. Both x and y are measured in meters, and t is measured in seconds. It is known that $\frac{dy}{dt} = te^{t-3} - 1$.

a.) Find the speed of the particle at time $t = 3$ seconds.

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b.) Find the total distance traveled by the particle for $0 \le t \le 4$ seconds.

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c.) Find the time t, $0 \le t \le 4$, when the line tangent to the path of the particle is horizontal. Is the direction of motion of the particle toward the left or toward the right at that time? Give a reason for your answer.

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d.) There is a point with x-coordinate 5 through which the particle passes twice. Find each of the following.

i.) The two values of t when that occurs

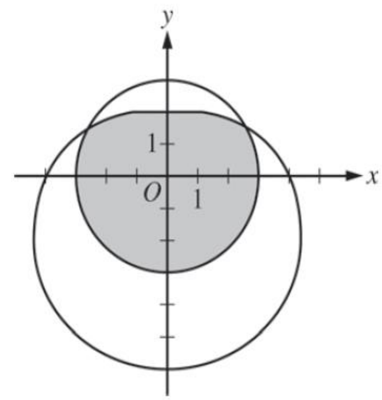
ii.) The slopes of the lines tangent to the particle's path at that point

iii.) The y-coordinate of that point, given $y(2) - 3 + \frac{1}{e}$

Polar #1
Calculator

Name____________________
Date____________Period__

The graphs of the polar curves $r = 3$ and $r = 4 - 2sin\theta$ are shown in the figure.
The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.



a.) Let S be the shaded region that is inside the graph of $r = 3$ and also outside the graph of $r = 4 - 2sin\theta$. Find the area of S.

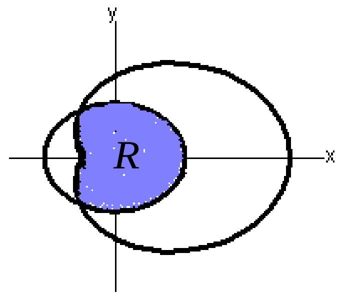
b.) A particle moves along the polar curve $r = 4 - 2sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \leq t \leq 2$ for which the x −coordinate of the particle's position is -1.

c.) For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time $t = 1.5$.

Polar #2
Calculator

Name___________________
Date____________Period___

The graphs of the polar curves $r = 2$ and $r = 3 + 2\cos\theta$ are shown in the figure.



a.) Let R be the region that is inside the graph of $r = 2$ and also inside the graph of $r = 3 + 2\cos\theta$, as shaded in the figure above. Find the area of R.

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b.) A particle moving with nonzero velocity along the polar curve given by $r = 3 + 2\cos\theta$ has position $\left(x(t), y(t)\right)$ at time t, with $\theta = 0$ when $t = 0$. This particle moves along the curve so that $\frac{dr}{dt} = \frac{dr}{d\theta}$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

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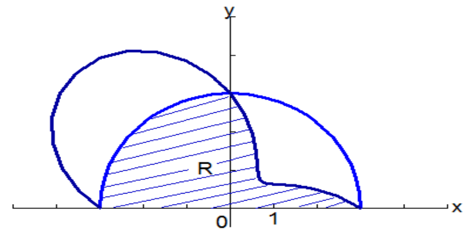
c.) For the particle described in part (b), $\frac{dy}{dt} = \frac{dy}{d\theta}$. Find the value of $\frac{dy}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

Polar #3 Name____________________

Calculator Date_______________Pd.___

The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure to the right for $0 \le \theta \le \pi$.



a.) Let R be the shaded region that is inside the graph of $r = 3$ and inside the graph of $r = 3 - 2\sin(2\theta)$. Find the area of R.

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b.) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$.

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c.) The distance between the two curves changes for $0 < \theta < \frac{\pi}{2}$. Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \frac{\pi}{3}$.

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d.) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \ge 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$.

Series #1 Name___________________
Non-Calculator Date____________Period___

The function g has derivatives of all orders, and the Maclaurin series for g is
$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \ldots$.

a.) Using the ratio test, determine the interval of convergence of the Maclaurin series for g.

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b.) The Maclaurin series for g evaluated at $x = \frac{1}{2}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $g\left(\frac{1}{2}\right)$ using the first tow nonzero terms of this series is $\frac{17}{120}$. Show that this approximation differs from $g\left(\frac{1}{2}\right)$ by less than $\frac{1}{200}$.

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c.) Write the first three nonzero terms and the general term of the Maclaurin series for $g'(x)$.

Series #2 Name___________________

Non-Calculator Date____________Period__

A function f has derivatives of all orders at $x = 0$. Let $P_n(x)$ denote the nth-degree Taylor polynomial for f about $x = 0$.

a.) It is known that $f(0) = -4$ and that $P_1\left(\frac{1}{2}\right) = -3$. Show that $f'(0) = 2$.

b.) It is known that $f''(0) = -\frac{2}{3}$ and $f'''(0) = \frac{1}{3}$. Find $P_3(x)$.

c.) The function h has first derivative given by $h'(x) = f(2x)$. It is known that $h(0) = 7$. Find the third-degree Taylor polynomial for h about $x = 0$.

Series #3 Name____________________
Non-Calculator Date_______________Pd.___

The Taylor series for a function f about $x = 1$ is given by $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2^n}{n}(x-1)^n$ and converges to $f(x)$ for $|x-1| < R$, where R is the radius of convergence of the Taylor series.

a.) Find the value of R.

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b.) Find the first three nonzero terms and the general term of the Taylor series for f' , the derivative of f, about $x = 1$.

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c.) The Taylor series for f' about $x = 1$, found in part (b), is a geometric series. Find the function f' to which the series converges for $|x-1| < R$. Use this function to determine f for $|x-1| < R$.

Practice Test #1 Name____________________

Calculator Problem 1 Date_______________Pd.___

The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time t=0.

a.) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

b.) Is the amount of water in the pipe increasing or decreasing at time t=3 hours? Give a reason for your answer.

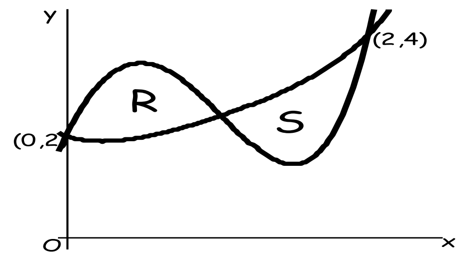
c.) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

d.) The pipe can hold 50 cubic feet of water before overflowing. For t>8, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, and equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

Practice Test #1 Name____________________

Calculator Problem 2 Date_______________Pd.___

Let *f* and *g* be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$ let *R* and *S* be the two regions enclosed by the graphs of *f* and *g* shown in the figure to the right.



a.) Find the sum of the areas of regions *R* and *S*.

b.) Region S is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

c.) Let *h* be the vertical distance between the graphs of *f* and *g* in region *S*. Find the rate of which *h* changes with respect to x when x=1.8.

Practice Test #1 Name____________________

Non-Calculator Problem 3 Date_______________Pd.___

Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differentiable function v, where *t* is measured in minutes and *v(t)* is measured in meters per minute, are given in the table.	t (minutes)	0	12	20	24	40
	v(t) (meters per min)	0	200	240	–220	150

a.) Use the data in the table to estimate the value of v'(16).

b.) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|dt$ in the context of the problem. Approximate the value of $\int_0^{40} |v(t)|dt$ using a right Riemann sum with the four subintervals indicated in the table.

c.) Bob is riding his bicycle along the same path. For $0 \le t \le 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where *t* is measured in minutes and *B(t)* is measured in meters per minute. Find Bob's acceleration at time *t*=5.

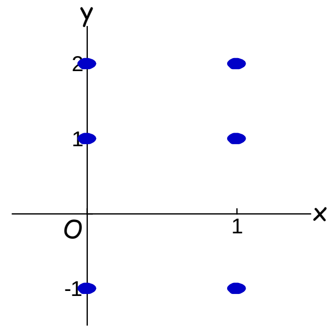
d.) Based on the model *B* from part (c), find Bob's average velocity during the interval $0 \le t \le 10$.

Practice Test #1
Non-Calculator Problem 4

Name____________________
Date_______________Pd.___

Consider the differential equation $\frac{dy}{dx} = 2x - y$

a.) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



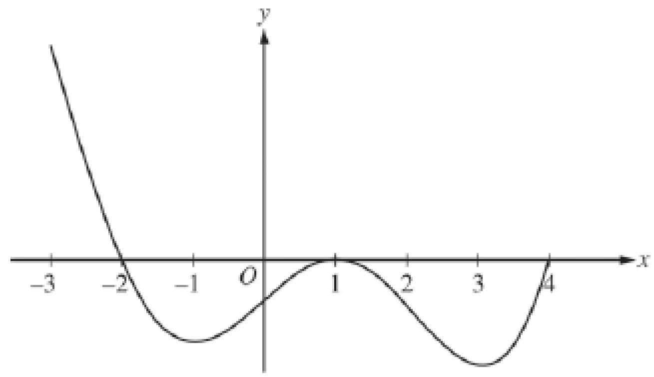
b.) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

c.) Let *y=f(x)* be the particular solution to the differential equation with the initial condition *f*(2)=3. Does *f* have a relative minimum, a relative maximum, or neither at x=2? Justify your answer.

d.) Find the values of the constants *m* and *b* for which *y=mx+b* is a solution to the differential equations.

Practice Test #1 Name____________________

Non-Calculator Problem 5 Date_______________Pd.___

The figure shows the graph of f', the derivative of a twice-differentiable function *f*, on the interval [-3,4]. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The area of the regions bounded by the x-axis and the graph of f' on the intervals [-2,1] and [1,4] are 9 and 12, respectively.	 Graph of f'

a.) Find all x-coordinates at which *f* has a relative maximum. Give a reason for your answer.

b.) On what open intervals contained in $-3 < x < 4$ is the graph of *f* both concave down and decreasing? Give a reason for your answer.

c.) Find the x-coordinates of all points of inflection for the graph of *f*. Give a reason for your answer.

d.) Given that *f*(1)=3, write an expression for *f(x)* that involves an integral. Find *f*(4) and *f*(-2).

Practice Test #1 Name_____________________
Non-Calculator Problem 6 Date________________Pd.___

Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$

a.) Write an equation for the line tangent to the curve at the point (-1,1).

b.) Find the coordinates of all points on the curve at which the line tangent to the curve at the point is vertical.

c.) Evaluate $\frac{d^2y}{dx^2}$ at the point on the curve where x=-1 and y=1.

Practice Test #2 Name____________________
Calculator Problem 1 Date_____________Pd_____

t (hours)	0	1	3	6	8
$R(t)$ (liters/hour)	1340	1190	950	740	700

Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-\frac{t^2}{20}}$ liters per hour for $0 \le t \le 8$, where *t* is measured in hours. Water is removed from the tank at a rated modeled by *R(t)* liters per hour, where *R* is differentiable and decreasing on $0 \le t \le 8$. Selected values of *R(t)* are shown in the table above. At *t=0*, there are 50,000 liters of water in the tank.

a.) Estimate R'(2). Show the work that leads to your answer. Indicate units of measure.

b.) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for you answer.

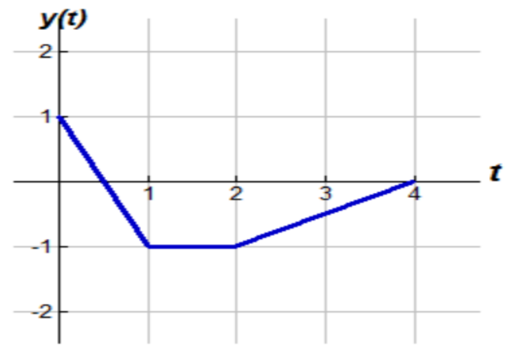
c.) Use your answer from part b.) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

d.) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not?

Practice Test #2
Calculator Problem 2

Name___________________
Date_____________Pd_____

At time *t*, the position of a particle moving in the *xy*-plane is given by the parametric functions $(x(t), y(t))$, where $\frac{dx}{dt} = t^2 + sin(3t^2)$. The graph of *y*, consisting of three line segments, is shown in the figure to the right. At *t*=0, the particle is at position (5,1).



a.) Find the position of the particle at *t*=3.

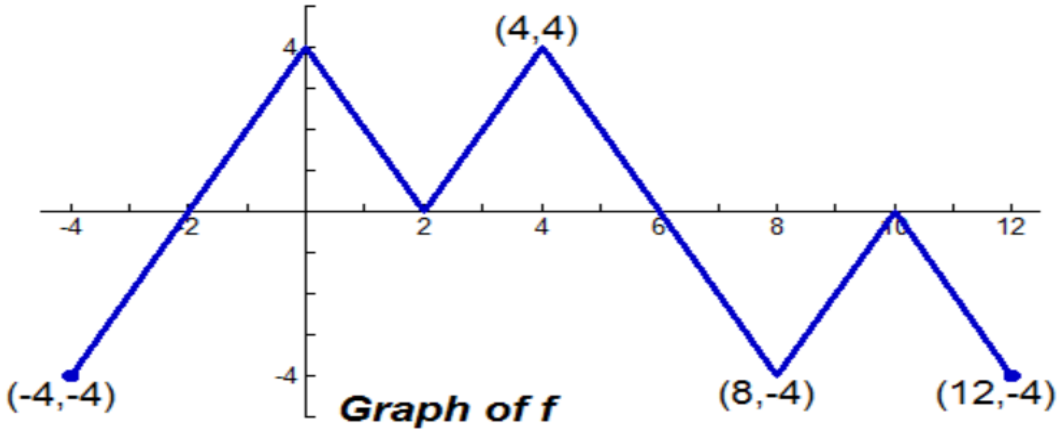
b.) Find the slope of the line tangent to the path of the particle at *t*=3.

c.) Find the speed of the particle at *t*=3.

d.) Find the total distance traveled by the particle from *t*=0 to *t*=2.

Practice Test #2
Non-Calculator Problem 3

Name___________________
Date____________Pd_____



The figure above shows the graph of the piecewise-linear function *f*. For $-4 \le x \le 12$, the function *g* is defined by $g(x) = \int_2^x f(t)dt$.

a.) Does *g* have a relative minimum, a relative maximum, or neither at x=*10*? Justify your answer.

b.) Does the graph of *g* have a point of inflection at x=*4*? Justify your answer?

c.) Find the absolute minimum value and the absolute maximum value of *g* on the interval $-4 \le x \le 12$? Justify your answer.

d.) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

Practice Test #2 Name___________________
Non-Calculator Problem 4 Date_____________Pd_____

Consider the differential equation $\frac{dy}{dx} = x^2 - \frac{1}{2}y$.

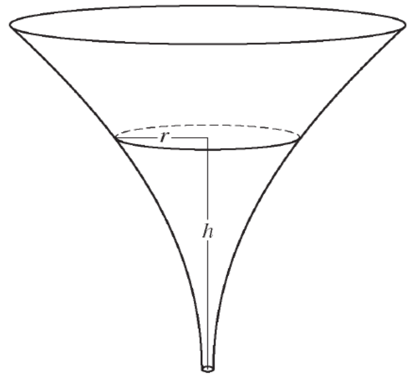
a.) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

b.) Let $y = f(x)$ be the particular solution to the given differential equation whose graph passes through the point (-2,8). Does the graph of *f* have a relative minimum, a relative maximum, or neither at the point (-2,8)? Justify your answer.

c.) Let $y = g(x)$ be the particular solution to the given differential equation with *g*(-1)=2. Find $\lim_{x \to -1} \frac{g(x)-2}{3(x+1)^2}$ Show the work that leads to your answer.

d.) Let $y = h(x)$ be a particular solution to the given differential equation with *h*(0)=2. Use Euler's method, starting at x=*0* with two steps of equal size, to approximate *h*(1).

Practice Test #2 Name___________________
Non-Calculator Problem 5 Date_____________Pd_____

The inside of a funnel of height 10 inches has a circular cross sections, as shown in the figure to the right. At height h, the radius of the funnel is given by $r = \frac{1}{20}(3 + h^2)$, where $0 \le h \le 10$. The units of *r* and *h* are inches.	

a.) Find the average value of the radius of the funnel.

b.) Find the volume of the funnel.

c.) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is *h*=3 inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

Practice Test #2 Name___________________

Non-Calculator Problem 6 Date_____________Pd_____

The function f has a Taylor series about x=1 that converges to f(x) for all x in the interval of convergence. It is known that $f(1) = 1$, $f'(1) = -\frac{1}{2}$, and the nth derivative of *f* at x=1 is given by $f^n(1) = (-1)^n \frac{(n-1)!}{2^n}$ for $n \geq 2$.

a.) Write the first four nonzero terms and the general term of the Taylor series for *f* about x=1.

b.) The Taylor series for *f* about x=1 has a radius of convergence of 2. Find the interval of convergence. Show the work that leads to your answer.

c.) The Taylor series for *f* about x=1 can be used to represent *f*(1.2) as an alternating series. Use the first three nonzero terms of the alternating series to approximate *f*(1.2).

d.) Show that the approximation found in part c.) is within 0.001 of the exact value of *f*(1.2).

Practice Test #3
Calculator Problem 1

Name___________________
Date_____________Pd_____

h (feet)	0	2	5	10
A(h) (square feet)	50.3	14.4	6.5	2.9

A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height *h* feet is given by the function *A*, where *A(h)* is measured in square feet. The function *A* is continuous and decreases as *h* increases. Selected values for *A(h)* are given in the table above.

a.) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure.

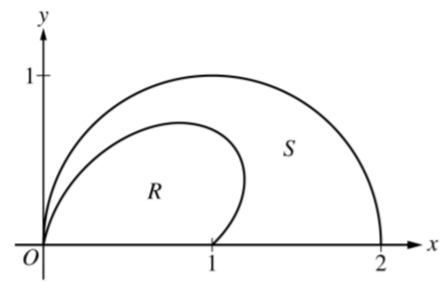
b.) Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.

c.) The area, in square feet, of the horizontal cross section at height *h* feet is modeled by the function *f* given by $f(h) = \frac{50.3}{e^{0.2h}+h}$. Based on this model, find the volume of the tank. Indicate units of measure.

d.) Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.

Practice Test #3 | Name___________________
Calculator Problem 2 | Date_____________Pd_____

The figure to the right shows the polar curves $r = f(\theta) = 1 + sin\theta\cos(2\theta)$ and $r = g(\theta) = 2cos\theta$ for $0 \le \theta \le \frac{\pi}{2}$. Let *R* be the region in the first quadrant bounded by $r = f(\theta)$ and the x-axis. Let *S* be the region in the first quadrant bounded by the curve $r = f(\theta)$, the curve $r = g(\theta)$, and the x-axis.



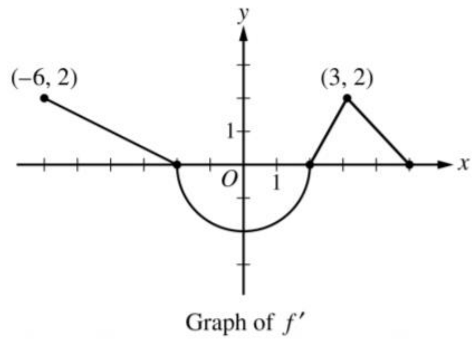
a.) Find the area of *R*.

b.) The ray $\theta = k$, where $0 \le k \le \frac{\pi}{2}$, divides *S* into two regions of equal area. Write but do not solve, an equation involving one or more integrals whose solution gives the value of *k*.

c.) For each θ, $0 \le \theta \le \frac{\pi}{2}$, let $w(\theta)$ be the distance between the points with polar coordinates $(f(\theta), \theta)$ and $(g(\theta), \theta)$. Write an expression for $w(\theta)$. Find w_A, the average value of $w(\theta)$ over the interval $0 \le \theta \le \frac{\pi}{2}$.

d.) Using the information in part (c), find the value of θ for which $w(\theta) = w_A$. Is the function $w(\theta)$ increasing or decreasing at that value of θ? Give a reason for your answer.

Practice Test #3 Name___________________
Non-Calculator Problem 3 Date_____________Pd_____

The function f is differentiable on the closed interval $[-6,5]$ and satisfies $f(-2)=7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure to the right.	

a.) Find the values of $f(-6)$ and $f(5)$.

b.) On what intervals is f increasing? Justify your answer.

c.) Find the absolute minimum value of f on the closed interval $[-6,5]$. Justify your answer.

d.) For each of $f''(-5)$ and $f''(3)$, find the value or explain why id does not exist.

Name___________________
Date_____________Pd_____

At time $t = 0$, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The interval temperature of the potato is 91 degrees Celcius (℃) at time $t = 0$, and the internal temperature of the potato is greater than 27℃ for all times $t > 0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$, where $H(t)$ is measure in degrees Celsius and $H(0) = 91$

a.) Write an equation for the line tangent to the graph of H at $t = 0$. Use this to approximate the internal temperature of the potato at time $t = 3$.

---

b.) Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate of an overestimate of the internal temperature of the potato at time $t = 3$.

---

c.) For $t > 10$, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\frac{dG}{dt} = -(G - 27)^{2/3}$, where $G(t)$ is measured in degrees Celsius and $G(0) = 91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the potato at time $t = 3$?

Practice Test #3 Name___________________

Non-Calculator Problem 5 Date_____________Pd_____

Let f be the function defined by $f(x) = \frac{3}{2x^2-7x+5}$.

a.) Find the slope of the line tangent to the graph of f at $x = 3$.

b.) Find the x-coordinate of each critical point of f in the interval $1 < x < 2.5$. Classify each critical point as the location of a relative minimum, a relative maximum, or neither. Justify your answers.

c.) Using the identity that $\frac{3}{2x^2-7x+5} = \frac{2}{2x-5} - \frac{1}{x-1}$, evaluate $\int_5^\infty f(x)dx$ or show the integral diverges.

d.) Determine whether the series $\sum_{n=5}^{\infty} \frac{3}{2x^2-7x+5}$ converges or diverges. State the conditions of the test used for determining converge or divergence.

Practice Test #3 Name___________________
Non-Calculator Problem 6 Date_____________Pd_____

The function f has derivatives of all orders for $-1 < x < 1$. The derivatives of f satisfy the conditions to the right. The Maclaurin series for f converges to $f(x)$ for $\lvert x\rvert < 1$.	$f(0) = 0$ $f'(0) = 1$ $f^{(n+1)}(0) = -n \cdot f^{(n)}(0)$ for all $n \geq 1$

a.) Show that the first four nonzero terms of the Maclaurin series for f are $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$, and write the general terms of the Maclaurin series for f.

---

b.) Determine whether the Maclaurin series described in part (a) converges absolutely, converges conditionally, or diverges at $x = 1$. Explain your reasoning.

---

c.) Write the first four nonzero terms and the general term of the Maclaurin series for $g(x) = \int_0^x f(t)dt$.

---

d.) Let $P_n\left(\frac{1}{2}\right)$ represent the nth degree Taylor polynomial for g about $x = 0$ evaluated at $x = \frac{1}{2}$, where g is the function defined in part (c). Use the alternating series error bound to show that $\left|P_4\left(\frac{1}{2}\right) - g\left(\frac{1}{2}\right)\right| < \frac{1}{500}$.

Practice Test #4 Name___________________
Calculator Problem 1 Date_____________Pd_____

People enter a line for an escalator at a rate modeled by the function *r* given by

$$r(t)=\begin{cases}44\left(\dfrac{t}{100}\right)^3\left(1-\dfrac{t}{300}\right)^7 & \text{for} \quad 0\le t\le 300\\ 0 & \text{for} \quad t>300\end{cases}$$

Where *r(t)* is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in the line at time *t=0*.

a.) How many people enter the line for the escalator during the time interval $0\le t\le 300$?

---

b.) During the time interval $0\le t\le 300$, there are always people in line for the escalator. How many people are in line at time *t=300*?

---

c.) For $t>300$, what is the first time t that there are no people I line for the escalator?

---

d.) For $0\le t\le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

Practice Test #4 Name___________________
Calculator Problem 2 Date_____________Pd_____

Researchers on a boat are investigating plankton cells in a sea. At a depth of h meters, the density of plankton cells, in millions of cells per cubic meter, is modeled by $p(h)=0.2h^2e^{-0.0025h^2}$ for $0\le h\le 30$ and is modeled by *f(h)* for $h\ge 30$. The continuous function *f* is not explicitly given.

a.) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.

---

b.) Consider a vertical column of water in the sea with horizontal cross section of constant area 3 square meters. To the nearest million, how many plankton cells are in the column of water between *h=0* and *h=30* meters?

---

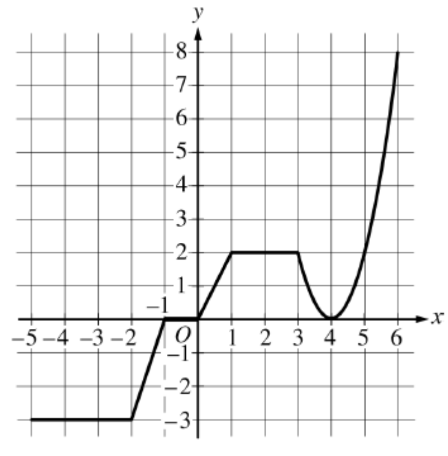
c.) There is a function *u* such that $0\le f(h)\le u(h)$ for all $h\ge 30$ and $\int_{30}^{\infty}u(h)dh=105$. The column of water in part (b) is K meters deep, where $K>30$. Write and expression involving one of more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.

---

d.) The boat is moving on the surface of the sea. At time $t\ge 0$, the position of the boat is $\left(x(t),y(t)\right)$, where $x'(t)=662\sin(5t)$ and $y'(t)=880\cos(6t)$. Time *t* is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0\le t\le 1$.

Practice Test #4 Name___________________
Non-Calculator Problem 3 Date_____________Pd_____

The graph of the continuous function g, the derivative of f, is shown in the figure to the right. The function is piecewise linear for $-5 \le x < 3$, $g(x) = 2(x-4)^2$ and for $3 \le x \le 6$.



Graph of g

a.) If $f(1) = 3$, what is the value of $f(-5)$?

b.) Evaluate $\int_1^6 g(x)dx$.

c.) For $-5 < x < 6$, on what open intervals, if any, is the graph of *f* both increasing and concave up? Give a reason for your answer.

d.) Find the x-coordinate of each point of inflection of the graph of *f*. Give a reason for yoru answer.

Practice Test #4 Name___________________
Non-Calculator Problem 4 Date_____________Pd_____

t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

The height of a tree at time *t* is given by a twice-differentiable function *H*, where *H(t)* is measured in meters and *t* is measured in years. Selected values of *H(t)* are given in the table above.

a.) Use the data in the table to estimate $H'(6)$. Using the correct units, interpret the meaning of $H'(6)$ in the context of the problem.

b.) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

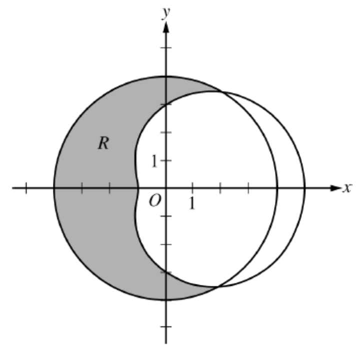
c.) Use a trapezoidal sum with four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$.

d.) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \dfrac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree si 50 meters tall, the diameter fo the base of the tree is increasing at a rate of 0.03 meters per year. According to this model, what is the rate of change of the height of the tree with respect t time, in meters per year, at the time when the tree is 50 meters tall?

Practice Test #4 Name___________________

Non-Calculator Problem 5 Date_____________Pd_____

The graphs of the polar curves $r=4$ and $r = 3 + 2\cos(\theta)$ are shown in the figure to the right. The curves intersect at $\theta = \frac{\pi}{3}$ and $\theta = \frac{5\pi}{3}$.



a.) Let R be the region that is inside the graph of $r = 4$ and also outside the graph of $r = 3 + 2\cos(\theta)$, as shown in the figure above. Write an expression involving an integral for the area of R.

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b.) Find the slope of the line tangent to the graph of $r = 3 + 2\cos(\theta)$ at $\theta = \frac{\pi}{2}$.

---

c.) A particle moves along the portion of the curve $r = 3 + 2\cos(\theta)$ for $0 < \theta < \frac{\pi}{2}$. The particle moves in such a way that the distance between the particle and the origin increases at a constant rate of 3 units per second. Find the rate at which the angle θ changes with respect to time at the instant when the position of the particle corresponds to $\theta = \frac{\pi}{3}$. Indicate the unit of measure.

Practice Test #4 Name___________________
Non-Calculator Problem 6 Date_____________Pd_____

The Maclaurin series for $\ln(1+x)$ is given by

$$x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+....+(-1)^{n+1}\frac{x^n}{n}+...$$

On its interval of convergence, this series converges to $\ln(1+x)$. Let f be the function defined by $f(x)=x\ln\left(1+\frac{x}{3}\right)$.

a.) Write the first four nonzero terms and the general term of the Maclaurin series for f.

b.) Determine the interval of convergence of the Maclaurin series for f. Show the work that leads to your answer.

c.) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x=0$. Use the alternating series error bound to find an upper bound for $|P_4(2)-f(2)|$.

50 Multiple Choice

1. Find the instantaneous rate of change at the point indicated. $f(x) = 2\sqrt{x}$; x = 4.

A. 0 B. ¼ C. ½ D. 4 E. 8

---

2. $\lim_{x\to 0} \frac{\sin\left(x+\frac{\pi}{2}\right)-1}{x} =$

A. -1 B. 0 C. 1 D. $\frac{\pi}{2}$ E. undefined

---

3. Let $f(x) = \begin{cases} x-3 & if\ x \le 2 \\ 2x+1 & if\ x > 2 \end{cases}$. Which of the following statements are true about f?

I. f(2) exists
II. f is continuous at 2
III. $\lim_{x\to 2} f(x)$ exists

A. I only B. II only C. III only D. I and II E. I, II, and III

---

4. $\lim_{x\to\infty} \frac{x^4-3x^2-2x+5}{3x^4-3x^2+3x-1} =$

A. -2/3 B. 0 C. 1/3 D. 1 E. 3

---

5. $\lim_{x\to 0} \frac{\sin 3x}{x} =$

A. -3 B. 0 C. 1 D. 3 E. DNE

---

6. $\lim_{h\to 0} \frac{(2+h)^5-2^5}{h} =$

A. 0 B. 1 C. 32 D. 80 E. 160

---

7. If $f(x) = 2x\sqrt{x} - 3\sqrt{x} + \frac{1}{\sqrt{x}}$, then *f '(x)* =

A. $2\sqrt{x} - \frac{3}{\sqrt{x}} + \frac{1}{x\sqrt{x}}$ B. $3\sqrt{x} - \frac{3}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}}$ C. $3\sqrt{x} - \frac{3}{\sqrt{x}} + \frac{1}{2x\sqrt{x}}$

D. $\frac{4x-3}{\sqrt{x}}$ E. $2\sqrt{x} - 3 + \frac{1}{x\sqrt{x}}$

8. Given a function f such that $f(1) = 2$, $f'(1) = 4$, and $h(x) = \frac{x^2}{f(x)}$, what is the value of $h'(1)$?

A. -2 B. -1/2 C. 0 D. 1 E. 2

---

9. If $f(x) = \sqrt{4\sin x + 2}$, then $f'(0) =$

A. -2 B. 0 C. $\sqrt{2}$ D. $\frac{\sqrt{2}}{2}$ E. 1

---

10. If $f(x) = x^2 \sin(x) + 2x\cos(x)$, then $f'(x) =$

A. $2x\cos(x) - 2\sin(x)$ B. $2x + \cos(x) + 2 - \sin(x)$ C. $x^2\cos(x) + 4x\sin(x) - 2\cos(x)$

D. $x^2\cos(x) + 2\cos(x)$ E. $2x\sin(x) - 2\cos(x)$

---

11. If $y = xy + x^2 + 1$, then when $x = -1$, $\frac{dy}{dx}$ is

A. ½ B. -1/2 C. -1 D. -1 E. nonexistent

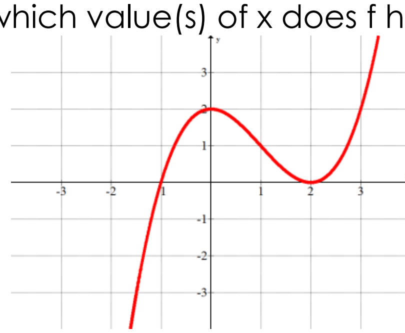
The function g is continuous on [-1, 2] and differentiable on (-1, 2). If g(-1) = 2 and g(2) = -4, which of the following statements is not necessarily true.

A. There exists a value c on (-1, 2) such that f(c) = 0.
B. There exists a value c on (-1, 2) such that f '(c) = 0.
C. There exists a value c on (-1, 2) such that f(c) = -3.
D. There exists a value c on (-1, 2) such that f '(c) = -2.
E. There exists a value c on [-1, 2] such that f(c) ≥ f(x) for all x on [-1, 2]

---

13. The graph shown depicts *f* ', the derivative of *f*. The graph of *f* ' has x-intercepts at -1 and 2 and a relative maximum where x = 0. At which value(s) of x does f have a point of inflection?

A. -1 only
B. -1 and 2
C. 0 only
D. 0 and 2
E. -1, 0, and 2



14. The function f given by f(x) = $3x^5 - 4x^3 - 3x$ has a relative maximum at x =

A. -1 B. $-\frac{\sqrt{5}}{5}$ C. 0 D. $\frac{\sqrt{5}}{5}$ E. 1

---

15. The radius, r, of a circle is decreasing at a rate of 4 centimeters per minute. Find the rate of change of area, A, when the radius is 5.

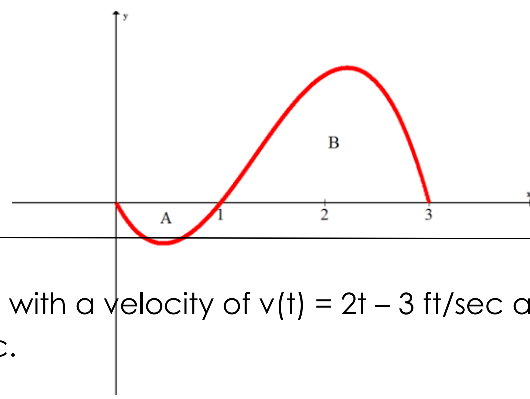
A. $\frac{dA}{dt} = -20\pi$ B. $\frac{dA}{dt} = -200\pi$ C. $\frac{dA}{dt} = 200\pi$ D. $\frac{dA}{dt} = -40\pi$ E. $\frac{dA}{dt} = 40\pi$

---

16. In the figure below, the area of region A is 2 and the area of region B is 11.

Find $\int_0^3 (f(x) - 2)dx$.

A. -3
B. 3
C. 7
D. 11
E. 15



---

17. If a particle is moving in a straight line with a velocity of v(t) = 2t – 3 ft/sec and its position at t = 2 seconds is -10ft, find its position at t = 5 sec.

A. -22 ft B. 2 ft C. 10 ft D. 12 ft E. 22ft

---

18. $\int_0^1 \sqrt{x}(x+1)dx =$

A. 0 B. 1 C. 16/15 D. 7/5 E. 2

---

19. If the substitution $u = \sqrt{x-1}$ is made, the integral $\int_2^5 \frac{\sqrt{x-1}}{x}dx =$

A. $\int_2^5 \frac{2u^2}{u^2+1}du$ B. $\int_1^2 \frac{u^2}{u^2+1}du$ C. $\int_1^2 \frac{u^2}{2(u^2+1)}du$

D. $\int_2^5 \frac{u}{u^2+1}du$ E. $\int_1^2 \frac{2u^2}{u^2+1}du$

20. $\int 2x^2(\sec(x^3))^2 dx = ?$

A. $-\frac{2}{3}\tan(x^3) + C$

B. $2\tan(x^3) + C$

C. $-2\tan(x^3) + C$

D. $\frac{2}{3}\tan(x^3) + C$

E. $6\tan(x^3) + C$

---

21. The region between the curves y = x^2 and y = $\sqrt{x}$ in the first quadrant is the base of a solid. For this solid, each cross section perpendicular to the y-axis is a rectangle whose height (above the xy-plane) is half of its length (in the xy-plane). Which of the following integrals gives the volume of this solid?

A. $\pi\int_0^1 (\sqrt{y} - y^2)^2 - \left(\frac{1}{2}(\sqrt{y} - y^2)\right)^2 dy$

B. $\frac{\pi}{2}\int_0^1 (\sqrt{y} - y^2)^2 dy$

C. $\int_0^1 (\sqrt{y} - y^2)^2 - \left(\frac{1}{2}(\sqrt{y} - y^2)\right)^2 dy$

D. $\int_0^1 (y - y^4) dy$

E. $\frac{1}{2}\int_0^1 (\sqrt{y} - y^2)^2 dy$

Calc 22. The area bounded by the curve $y = \frac{1}{2}x^2$, the x-axis, and the lines x = k and x = 2, where 0 < k < 2, is rotated about the x-axis. The volume of the solid formed is 5. Find the value of k.

A. 0.002

B. 0.701

C. 0.967

D. 1.565

E. 3.248

---

23. Find the volume of the solid formed by revolving the region bounded by y = 6 and $y = 12 - \frac{x^2}{12}$ about the x-axis.

A. $\frac{2016}{5}\sqrt{2}\pi$

B. $\frac{4032}{5}\sqrt{2}\pi$

C. $\frac{1008}{5}\sqrt{2}\pi$

D. $-\frac{2016}{5}\sqrt{2}\pi$

E. $-\frac{1008}{5}\sqrt{2}\pi$

Calc 24. Find the volume of the solid formed when the area in the first quadrant between f(x) = sin(2x) and g(x) = x is rotated about the line y = 2.

A. 0.057 B. 0.179 C. 0.576 D. 1.809 E. 3.479

---

25. $\frac{d}{dx}(xe^{2lnx}) =$

A. 2 B. 3 C. $3x^2$ D. $\frac{e^{2lnx}}{x}$ E. $2e^{2lnx}$

---

26. If f(x) = ln(2x – e^{-x} + 3), then f'(0) is

A. ½ B. 2/3 C. ln 2 D. 1 E. 1

---

27. If $\frac{dy}{dx} = ky$ where k is a nonzero constant, then y could be

A. $3e^{kxy}$ B. $3e^{kx}$ C. $e^{kx} + 3$ D. kx + 3 E. $ky^2 + k$

---

28. $\frac{d}{dx}\left(tan^{-1}(x)\right) =$

A. $\frac{1}{\sqrt{1-x^2}}$ B. $\frac{1}{x^2+1}$ C. $\frac{3}{x^2+1}$ D. $\frac{9}{x^2+9}$ E. $\frac{1}{x^2+9}$

---

29. What are ALL values of p for which $\int_1^\infty \frac{1}{x^{2p-1}}$ dx diverges?

A. p < ½ B. p > ½ C. p > 1 D. p < 1 E. all values

---

30. $\int_1^\infty \frac{x}{(1+x^2)^2}dx$ is

A. -1/2 B. -1/4 C. ¼ D. ½ E. divergent

31. $\int x^2 sinx dx =$

A. $-x^2\cos x - 2x\sin x - 2\cos x + C$

B. $-x^2\cos x + 2x\sin x - 2\cos x + C$

C. $-x^2\cos x + 2x\sin x + 2\cos x + C$

D. $-\frac{x^3}{3}cosx + C$

E. $2x\cos x + C$

---

32. The length of a curve from x = 2 to x = 3 is given by $\int_2^3 \sqrt{1+(12x^2-4)^2}dx$. Which of the following is the equation of the curve if the curve goes through the point (2, 22)?

A. $y = 4x^3 - 4x$

B. $y = 4x^3 - 4x + 2$

C. $y = 4x^3 - 4x - 2$

D. $y = 12x^2 - 4$

E. $y = 12x^2 - 94$

---

33. Given the third degree Taylor polynomial $5 - 2(x - 1) + 3(x - 1)^2 - 6(x - 1)^3$ of f(x), what is the value of
f ''(1)?

A. -36

B. -6

C. 0

D. 6

E. 36

---

34. Given f(0) = 2, f '(0) = 3, and f''(0) = 4 , which of the following is the second degree Maclaurin polynomial for f(x)?

A. $2 - 3x + 2x^2$

B. $2 - 3x + 4x^2$

C. $-2 + 3x - 2x^2$

D. $2 + 3x + 2x^2$

E. $2 + 3x + 4x^2$

35. Let f be the function given by f(x) = ln(3 – x). The third-degree Taylor polynomial for f about x = 2 is

A. $-(x-2)+\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$
B. $-(x-2)-\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$
C. $(x-2)+(x-2)^2+(x-2)^3$
D. $(x-2)+\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$
E. $(x-2)-\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

---

36. Let y = f(x) be the solution to the differential equation $\frac{dy}{dx}=\frac{x}{y^2}$ with initial condition f(0) = 1. Then f(x) =

A. 2x
B. $\sqrt[3]{3x^2+\frac{1}{3}}$
C. $\sqrt[3]{3x+1}$
D. $\sqrt[3]{3x^2+1}$
E. $\sqrt[3]{\frac{3}{2}x^2+1}$

---

37. The logistic equation $P(t)=\frac{400}{1-2e^{-0.4t}}$ represents the population of squirrels in a 2,000 acre forest. The population is growing fastest when the population is

A. 50
B. 100
C. 200
D. 300
E. 400

---

38. Which of the following series converges?

I. $\sum_{n=1}^{\infty}\frac{1}{n+2\sqrt{n}}$
II. $\sum_{n=1}^{\infty}\frac{n}{\sqrt{n^5-1}}$
III. $\sum_{n=1}^{\infty}\frac{\ln n}{n^2}$

A. I and II
B. II and III
C. I and III
D. II only
E. III only

39. For which of the following series is the Ratio Test inconclusive?

A. $\sum_{n=1}^{\infty}\frac{1}{n!}$ B. $\sum_{n=1}^{\infty}\frac{2^n}{n^2}$ C. $\sum_{n=1}^{\infty}\frac{3n}{2n^3+1}$ D. $\sum_{n=1}^{\infty}\frac{n!}{n^3}$ E. $\sum_{n=1}^{\infty}\frac{e^n}{(n-1)!}$

---

40. Which of the following series converges absolutely?

I. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{1.1^n}$

II. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n\cos(\pi n)}$

III. $\sum_{n=1}^{\infty}\frac{(-1)^n}{2^n+.5^n}$

A. I only   B. II only   C. III only   D. I and II   E. I and III

---

41. In the Maclaurin series expansion of $f(x) = 8(x + 4)^{3/2}$, what is the coefficient of the x^3 term?

A. -3   B. -3/8   C. -1/16   D. 3/8   E. 3/2

---

42. What are all values of x for which the series $\sum_{n=1}^{\infty}\frac{(x-2)^n}{n3^n}$ converges?

A. $-3 \le x \le 3$ B. $-3 < x < 3$ C. $-1 < x \le 5$ D. $-1 \le x \le 5$ E. $-1 \le x < 5$

---

43. Which of the following gives the slope of the line tangent to the graph of the relation given by the parametric equations $x = 4t^2 - 6t + 2$ and $y = -3t^2$?

A. $8t - 6$ B. $-6t$ C. $\frac{4t-3}{-3t}$ D. $\frac{-3t}{4t-3}$ E. $-\frac{3}{4}$

---

44. Which of the following gives the length of the path described by the parametric equations x = $\sin^2 t$
and y = tan(2t) from $x = \frac{\pi}{4}$ to $x = \frac{3\pi}{4}$?

A. $\int_{\pi/4}^{3\pi/4} \sqrt{sin^4 t + tan^2(2t)}dt$

B. $\int_{\pi/4}^{3\pi/4} \sqrt{4cos^2 t + 4sec^2(2t)}dt$

C. $\int_{\pi/4}^{3\pi/4} \sqrt{4sin^2 tcos^2 t + 4sec^4(2t)}dt$

D. $\int_{\pi/4}^{3\pi/4} \sqrt{4cos^2 t + 4sec^4(2t)}dt$

E. $\int_{\pi/4}^{3\pi/4} \sqrt{2sintcost + 2sec^2(2t)}dt$

---

45. If r(t) = <t^3, 2t – 1>, then r'(2) =

A. 4 B. $3\sqrt{17}$ C. <8, 3> D. <12, 2> E. <12, 3>

---

46. The path of a particle satisfies $\frac{dr}{dt} =< 4t^3, e^t >$. If r(0) = <1, 4>, then what is the location of r(2)?

A. <0, 1> B. <1, 3> C. <4, e^4> D. <17, e^2 + 3> E. <19, e^5>

---

47. Which of the following gives the area inside $r = 2 + 2sin\theta$ and outside $r = 4sin\theta$?

A. $\frac{1}{2}\int_{\pi/2}^{3\pi/2}((2 + 2sin\theta)^2 - (4sin\theta)^2)d\theta$

B. $\int_{\pi/2}^{3\pi/2}((2 + 2sin\theta)^2 - (4sin\theta)^2)d\theta$

C. $\int_{\pi/2}^{3\pi/2}((2 + 2sin\theta)^2 - (4sin\theta)\ \ d\theta$

D. $2\int_{\pi/2}^{3\pi/2}((2 + 2sin\theta)^2 - (4sin\theta)^2)d\theta$

E. $\int_{\pi/2}^{3\pi/2}((2 + 2sin\theta)^2 - (4sin\theta)\ \ d\theta$

48. If x = e^{2t} and y = sin(2t), then $\frac{dy}{dx}$ =

A. $4e^{2t}\cos(2t)$ B. $\frac{e^{2t}}{\cos(2t)}$ C. $\frac{\sin(2t)}{2e^{2t}}$ D. $\frac{\cos(2t)}{2e^{2t}}$ E. $\frac{\cos(2t)}{e^{2t}}$

---

49. For what values of t does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

A. 0 only   B. 1 only   C. 0 and 2/3   D. 0, 2/3, and 1   E. none

---

50. Find $\frac{dy}{dx}$. $x = t^2$ *and* $y = 3 - 2t$

A. $\frac{dy}{dx} = -\frac{2}{t}$ B. $\frac{dy}{dx} = -\frac{1}{t}$ C. $\frac{dy}{dx} = -\frac{t}{2}$ D. $\frac{dy}{dx} = -\frac{4}{t}$ E. $\frac{dy}{dx} = -\frac{4}{t^2}$

Multiple Choice #1
Non- Calculator

Name___________________
Date____________Period___

1. At time $t \geq 0$, a particle moving in the xy –plane has velocity vector given by $v(t) = \langle t^2, 5t \rangle$. What is the acceleration vector of the particle at time $t = 3$?

(A) $\langle 9, \frac{45}{2} \rangle$
(B) $\langle 6, 5 \rangle$
(C) $\langle 2, 0 \rangle$
(D) $\sqrt{306}$
(E) $\sqrt{61}$

2. $\int xe^{x^2}\, dx =$

(A) $\frac{1}{2}e^{x^2} + C$
(B) $e^{x^2} + C$
(C) $xe^{x^2} + C$
(D) $\frac{1}{2}e^{2x} + C$
(E) $e^{2x} + C$

3. $\lim_{x \to 0} \frac{sinxcosx}{x}$ is

(A) -1
(B) 0
(C) 1
(D) $\frac{\pi}{4}$
(E) $nonexistent$

4. Consider the series $\sum_{n=1}^{\infty} \frac{e^n}{n!}$. If the ratio test is applied to the series, which of the following inequalities results, implying that the series converges?

(A) $\lim_{n \to \infty} \frac{e}{n!} < 1$
(B) $\lim_{n \to \infty} \frac{n!}{e} < 1$
(C) $\lim_{n \to \infty} \frac{n+1}{e} < 1$
(D) $\lim_{n \to \infty} \frac{e}{n+1} < 1$
(E) $\lim_{n \to \infty} \frac{e}{(n+1)!} < 1$

5. Which of the following gives the length of the path described by the parametric equations $x = sin(t^3)$ and $y = e^{5t}$ from $t = 0$ to $t = \pi$?

(A) $\int_0^{\pi} \sqrt{sin^2(t^3) + e^{10t}}\, dt$

(B) $\int_0^{\pi} \sqrt{cos^2(t^3) + e^{10t}}\, dt$

(C) $\int_0^{\pi} \sqrt{9t^4 cos^2(t^3) + 25e^{10t}}\, dt$

(D) $\int_0^{\pi} \sqrt{3t^2 cos(t^3) + 5e^{5t}}\, dt$

(E) $\int_0^{\pi} \sqrt{cos^2(3t^2) + e^{10t}}\, dt$

6. Let f be the function defined. Which of the following statements about f are true?

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & if\ x \neq 2 \\ 1 & if\ x = 2 \end{cases}$$

I. f has a limit at $x = 2$.
II. f is continuous at $x = 2$.
III. f is differentiable at $x = 2$.

(A) I *only*
(B) II *only*
(C) III *only*
(D) I *and* II *only*
(E) I, II, *and* III *only*

7. Given that $y(1) = -3$ and $\frac{dy}{dx} = 2x + y$, what is the approximation for $y(2)$ if Euler's method is used with a step size of 0.5, starting at $x = 1$?

(A) -5
(B) -4.25
(C) -4
(D) -3.75
(E) -3.5

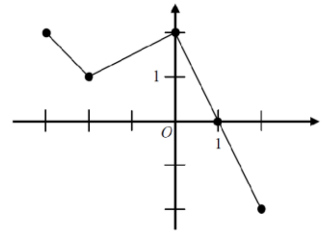
x	2	3	5	8	13
$f(x)$	6	-2	-1	3	9

8. The function f is continuous on the closed interval [2,13] and has values as shown in the table above. Using the intervals [2,3], [3,5], [5,8], and [8,13], what is the approximation of $\int_2^{13} f(x)dx$ obtained from a left Riemann sum?

(A) 6
(B) 14
(C) 28
(D) 32
(E) 50

9. The graph of the piecewise linear function f is shown in the figure. If $g(x) = \int_{-2}^{x} f(t)dt$, which of the following values is greatest?

(A) $g(-3)$
(B) $g(-2)$
(C) $g(0)$
(D) $g(1)$
(E) $g(2)$



Graph of f

10. In the xy −plane, what is the slope of the line tangent to the graph of $x^2 + xy + y^2 = 7$ at the point $(2,1)$?

(A) $-\frac{4}{3}$
(B) $-\frac{5}{4}$
(C) -1
(D) $-\frac{4}{5}$
(E) $-\frac{3}{4}$

11. Let R be the region between the graph of $y = e^{-2x}$ and the x −axis for $x \geq 3$. The area of R is

(A) $\frac{1}{2e^6}$
(B) $\frac{1}{e^6}$
(C) $\frac{2}{e^6}$
(D) $\frac{\pi}{2e^6}$
(E) Infinite

12. Which of the following series converges for all real numbers x?

(A) $\sum_{n=1}^{\infty} \frac{x^n}{n}$
(B) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$
(C) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$
(D) $\sum_{n=1}^{\infty} \frac{e^n x^n}{n!}$
(E) $\sum_{n=1}^{\infty} \frac{n!\, x^n}{e^n}$

13. $\int_1^e \frac{x^2+1}{x}dx =$

(A) $\frac{e^2-1}{2}$

(B) $\frac{e^2+1}{2}$

(C) $\frac{e^2+2}{2}$

(D) $\frac{e^2-1}{e^2}$

(E) $\frac{2e^2-8e+6}{3e}$

x	0	1	2	3
$f''(x)$	5	0	-7	4

14. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0,2)$.

(B) f is decreasing on the interval $(0,2)$.

(C) f has a local maximum at $x = 1$.

(D) The graph of f has a point of inflection at $x = 1$.

(E) The graph of f changes concavity in the interval $(0,2)$.

15. If $f(x) = (lnx)^2, then\ f''(\sqrt{e}) =$

(A) $\frac{1}{e}$

(B) $\frac{2}{e}$

(C) $\frac{1}{2\sqrt{e}}$

(D) $\frac{1}{\sqrt{e}}$

(E) $\frac{2}{\sqrt{e}}$

16. What are all values of x for which the series $\sum_{n=1}^{\infty}\left(\frac{2}{x^2+1}\right)^n$ converge?

(A) $-1 < x < 1$

(B) $x > 1$ only

(C) $x \geq 1$ only

(D) $x < -1$ and $x > 1$ only

(E) $x \leq -1$ and $x \geq 1$

17. Let h be a differentiable function, and let f be the function defined by $f(x) = h(x^2 - 3)$. Which of the following is equal to $f'(2)$?

(A) $h'(1)$
(B) $4h'(1)$
(C) $4h'(2)$
(D) $h'(4)$
(E) $4h'(4)$

18. In the xy –plane, the line $x + y = k$, where k is a constant, is tangent to the graph of $y = x^2 + 3x + 1$. What is the value of k?

(A) -3
(B) -2
(C) -1
(D) 0
(E) 1

19. $\int \frac{7x}{(2x-3)(x+2)} dx =$

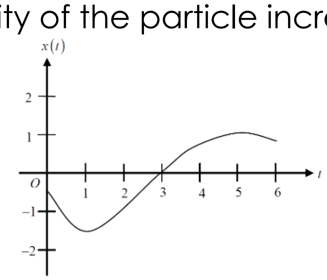
(A) $\frac{3}{2} ln|2x - 3| + 2ln|x + 2| + C$
(B) $3ln|2x - 3| + 2ln|x + 2| + C$
(C) $3ln|2x - 3| - 2ln|x + 2| + C$
(D) $-\frac{6}{(2x-3)^2} - \frac{2}{(x+2)^2} + C$
(E) $-\frac{3}{(2x-3)^2} - \frac{2}{(x+2)^2} + C$

20. What is the sum of the series $1 + ln2 + \frac{(ln2)^2}{2!} + \cdots + \frac{(ln2)^n}{n!} + \cdots$?

(A) $ln2$
(B) $ln\,(1 + ln2)$
(C) 2
(D) e^2
(E) $The\ series\ diverges$

21. A particle moves along a straight line. The graph of the particles position $x(t)$ at time t is shown for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

(A) $0 < t < 2$
(B) $1 < t < 5$
(C) $2 < t < 6$
(D) $3 < t < 5\ only$
(E) $1 < t < 2\ and\ 5 < t < 6$



22. The table gives values of $f, f', g,$ and g' for selected values of x.

If $\int_0^1 f'(x)g(x)dx = 5$, then $\int_0^1 f(x)g'(x)dx =$

(A) -14
(B) -13
(C) -2
(D) 7
(E) 15

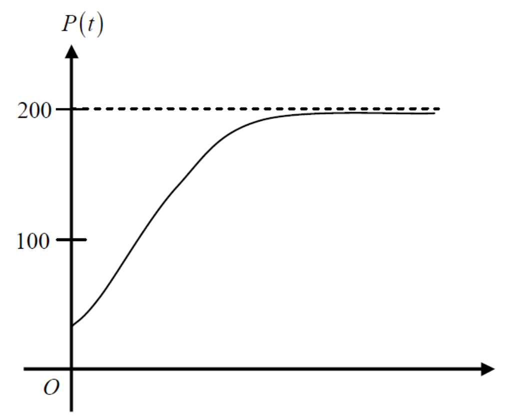
x	0	1
$f(x)$	2	4
$f'(x)$	6	-3
$g(x)$	-4	3
$g'(x)$	2	-1

23. If $f(x) = x\sin(2x)$, which of the following is the Taylor series for f about $x = 0$?

(A) $x - \frac{x^3}{2!} + \frac{x^5}{4!} - \frac{x^7}{6!} + \cdots$
(B) $x - \frac{4x^3}{2!} + \frac{16x^5}{4!} - \frac{64x^7}{6!} + \cdots$
(C) $2x - \frac{8x^3}{3!} + \frac{32x^5}{5!} - \frac{128x^7}{7!} + \cdots$
(D) $2x^2 - \frac{2x^4}{3!} + \frac{2x^6}{5!} - \frac{2x^8}{7!} + \cdots$
(E) $2x^2 - \frac{8x^4}{3!} + \frac{32x^6}{5!} - \frac{128x^8}{7!} + \cdots$

24. Which of the following differential equations for a population P could model the logistic growth shown in the figure?

(A) $\frac{dP}{dt} = 0.2P - 0.001P^2$
(B) $\frac{dP}{dt} = 0.1P - 0.001P^2$
(C) $\frac{dP}{dt} = 0.2P^2 - 0.001P$
(D) $\frac{dP}{dt} = 0.1P^2 - 0.001P$
(E) $\frac{dP}{dt} = 0.1P^2 + 0.001P$



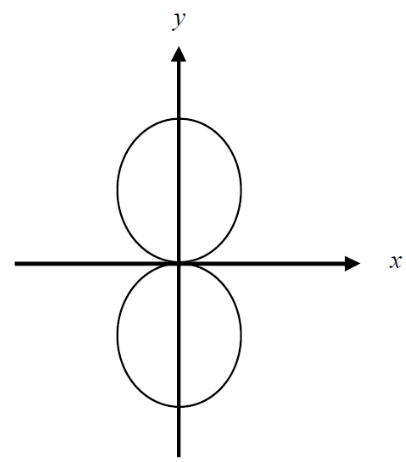
25. Let f be the function defined below, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?

$$f(x) = \begin{cases} cx + d & for\ \ x \leq 2 \\ x^2 - cx & for\ x > 2 \end{cases}$$

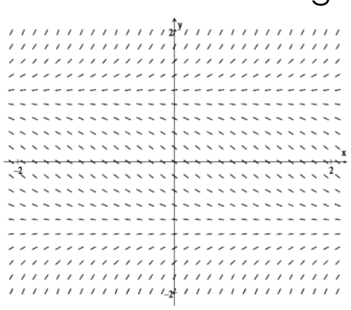
(A) -4
(B) -2
(C) 0
(D) 2
(E) 4

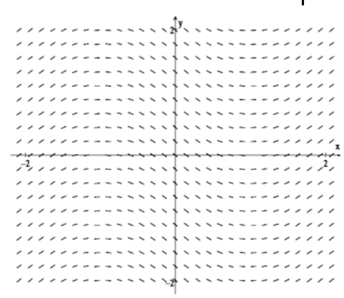
26. Which of the following expressions gives the total area enclosed by the polar curve $r = sin^2\theta$ shown in the figure?

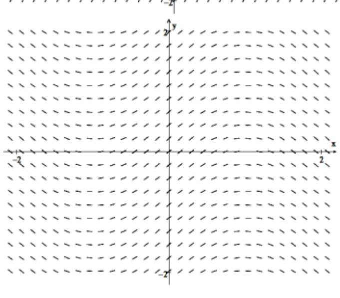
(A) $\frac{1}{2}\int_0^{\pi} sin^2\theta\, d\theta$
(B) $\int_0^{\pi} sin^2\theta\, d\theta$
(C) $\frac{1}{2}\int_0^{\pi} sin^4\theta\, d\theta$
(D) $\int_0^{\pi} sin^4\theta\, d\theta$
(E) $2\int_0^{\pi} sin^4\theta\, d\theta$

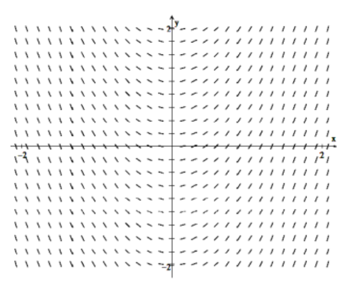


27. Which of the following could be the slope field for the differential equation $\frac{dy}{dx} = y^2 - 1$?

A


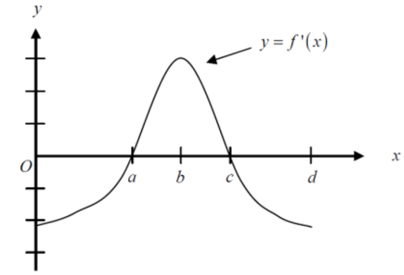
B


C


D


Multiple Choice #1 Name___________________
Calculator Date____________Period___

1. The graph of f', the derivative of a function f, is shown. The domain of f is the open interval $0 < x < d$. Which of the following statements is true?

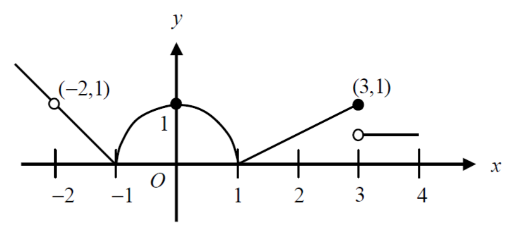


(A) $f\ has\ a\ local\ minimum\ at\ x = c$
(B) $f\ has\ a\ local\ maximum\ at\ x = b$
(C) $The\ graph\ of\ f\ has\ a\ point\ of\ inflection\ at\ (a, f(a))$
(D) $The\ graph\ of\ f\ has\ a\ point\ of\ inflection\ at\ (b, f(b))$
(E) $The\ graph\ of\ f\ is\ concave\ up\ on\ the\ open\ interval\ (c, d)$

2. Water is pumped out of a lake at the rate $R(t) = 12\sqrt{\frac{t}{t+1}}$ cubic meters per minute, where t is measured in minutes. How much water is pumped from time $t = 0$ to $t = 5$?

(A) 9.439 cubic meters
(B) 10.954 cubic meters
(C) 43.816 cubic meters
(D) 47.193 cubic meters
(E) 54.72 cubic meters

3. The graph of a function f is shown. For which of the following values of c does $\lim_{x \to c} f(x) = 1$?



Graph of f

(A) $0\ only$
(B) $0\ and\ 3\ only$
(C) $-2\ and\ 0\ only$
(D) $-2\ and\ 3\ only$
(E) $-2, 0, and\ 3$

4. Let f be a positive, continuous, decreasing function such that $a_n = f(n)$. If $\sum_{n=1}^{\infty} a_n$ converges to k, which of the following must be true?

(A) $\lim_{n \to \infty} a_n = k$
(B) $\int_1^n f(x)dx = k$
(C) $\int_1^{\infty} f(x)dx\ \ diverges$
(D) $\int_1^{\infty} f(x)dx\ \ converges$
(E) $\int_1^{\infty} f(x)dx = k$

5. The derivative of the function f is given by $f'(x) = x^2\cos(x^2)$. How many points of inflection does the graph of f have on the open interval $(-2,2)$?

(A) One
(B) Two
(C) Three
(D) Four
(E) Five

6. Let f and g be continuous for $a \leq x \leq b$. If $a < c < b$, $\int_a^b f(x)dx = P$, $\int_c^b f(x)dx = Q$, $\int_a^b g(x)dx = R$, and $\int_c^b g(x)dx = S$, then $\int_a^c (f(x) - g(x))dx =$?

(A) $P - Q + R - S$
(B) $P - Q - R + S$
(C) $P - Q - R - S$
(D) $P + Q - R - S$
(E) $P + Q - R + S$

7. If $\sum_{n=1}^{\infty} a_n$ diverges and $0 \leq a_n \leq b_n$ for all n, which of the following statements must be true?

(A) $\sum_{n=1}^{\infty}(-1)^n a_n$ converges
(B) $\sum_{n=1}^{\infty}(-1)^n b_n$ converges
(C) $\sum_{n=1}^{\infty}(-1)^n b_n$ diverges
(D) $\sum_{n=1}^{\infty} b_n$ converges
(E) $\sum_{n=1}^{\infty} b_n$ diverges

8. What is the area enclosed by the curves of $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$?

(A) 10.667
(B) 11.833
(C) 14.583
(D) 21.333
(E) 32

9. Let f be a function with $f(3) = 2$, $f'(3) = -1$, $f''(3) = 6$, and $f'''(x) = 12$. Which of the following is the third-degree Taylor polynomial for f about $x = 3$?

(A) $2 - (x-3) + 3(x-3)^2 + 2(x-3)^3$
(B) $2 - (x-3) + 3(x-3)^2 + 4(x-3)^3$
(C) $2 - (x-3) + 6(x-3)^2 + 12(x-3)^3$
(D) $2 - x + 3x^2 + 2x^3$
(E) $2 - x + 6x^2 + 12x^3$

10. A particle moves on the x –axis with velocity given by $v(t) = 3t^4 - 11t^2 + 9t - 2$ for $-3 \leq t \leq 3$. How many times does the particle change direction as t increases from –3 to 3?

(A) $Zero$
(B) One
(C) Two
(D) Three
(E) Four

11. On the graph of $y = f(x)$, the slope at any point (x, y) is twice the value of x. If $f(2) = 3$, what is the value of $f(3)$?

(A) 6
(B) 7
(C) 8
(D) 9
(E) 10

12. An object traveling in a straight line has position $x(t)$ at time t. If the initial position is $x(0) = 2$ and the velocity of the object is $v(t) = \sqrt[3]{1 + t^2}$, what is the position of the object at time $t = 3$?

(A) 0.431
(B) 2.154
(C) 4.512
(D) 6.512
(E) 17.408

13. For all values of x, the continuous function f is positive and decreasing. Let g be the function given by $g(x) = \int_2^x f(t)dt$. Which of the following could be a table of values for g?

A)

x	$g(x)$
1	-2
2	0
3	1

B)

x	$g(x)$
1	-2
2	0
3	3

C

x	$g(x)$
1	1
2	0
3	-2

D)

x	$g(x)$
1	2
2	0
3	-1

E)

x	$g(x)$
1	3
2	0
3	2

14. The function f is continuous for $-2 \leq x \leq 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following statements must be true?

(A) $For -2 < k < 2,\ f'(k) > 0$
(B) $For -2 < k < 2,\ f'(k) < 0$
(C) $For -2 < k < 2,\ f'(k)\ exists$
(D) $For -2 < k < 2,\ f'(k)\ exists, but\ f'is\ not\ continuous.$
(E) $For\ some\ k, where\ -2 < k < 2, f'(k) does\ not\ exist.$

15. The table gives the values of the differentiable functions f and g and of their derivatives f' and g', at selected values of x. If $h(x) = f(g(x))$, what is the slope of the graph of h at $x = 2$?

(A) -10
(B) -6
(C) 5
(D) 6
(E) 10

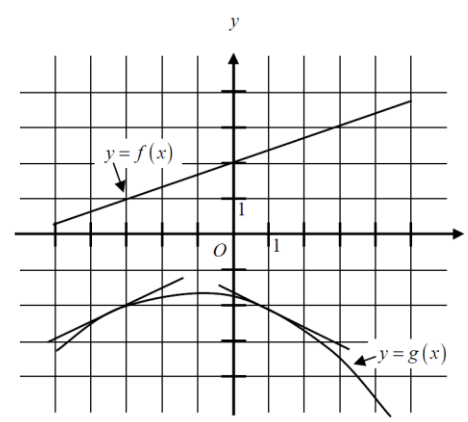
x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	-5	1	3	0
0	-2	0	1	1
1	0	-3	0	0.5
2	5	-1	5	2

16. Let f be the function given by $f(x) = \int_{1/3}^{x} cos\left(\frac{1}{t^2}\right) dt$ for $\frac{1}{3} \leq x \leq 1$. At which of the following values of x does f attain a relative maximum?

(A) 0.357 and 0.798
(B) 0.4 and 0.564
(C) 0.4 only
(D) 0.461
(E) 0.999

17. The figure shows the graphs of the functions f and g. The graphs of the lines tangent to the graph of g at $x = -3$ and $x = 1$ are also shown. If $B(x) = g(f(x))$, what is $B'(-3)$?

(A) $-\frac{1}{2}$
(B) $-\frac{1}{6}$
(C) $\frac{1}{6}$
(D) $\frac{1}{3}$
(E) $\frac{1}{2}$



Multiple Choice #3 Name___________________
Non- Calculator Date____________Period___

1. If $y = \sin(3x)$ then $\frac{dy}{dx}$=

(A) $-3\cos(3\text{x})$

(B) $-\cos(3\text{x})$

(C) $-\frac{1}{3}\cos(3\text{x})$

(D) $\cos(3\text{x})$

(E) $3\cos(3\text{x})$

2. $\lim_{n\to 0} \frac{e^x - cosx - 2x}{x^2 - 2x}$ is

(A) $-\frac{1}{2}$

(B) 0

(C) $\frac{1}{2}$

(D) 1

(E) nonexistent

3. $\int (3x+1)^5\, dx =$

(A) $\frac{(3x+1)^6}{18} + C$

(B) $\frac{(3x+1)^6}{6} + C$

(C) $\frac{(3x+1)^6}{2} + C$

(D) $\frac{(\frac{3x^2}{2}+1)^6}{2} + C$

(E) $\left(\frac{3x^2}{2}+x\right)^5 + C$

4. For $0 \leq t \leq 13$ an object travels along an elliptical path given by the parametric equations $x = 3cost$ and $y = 4sint$. At the point where $t = 13$, the object leaves the path and travels along the line tangent to the path at the point. What is the slope of the line on which the object travels?

(A) $-\frac{4}{3}$

(B) $-\frac{3}{4}$

(C) $-\frac{4tan13}{3}$

(D) $-\frac{4}{3tan13}$

(E) $-\frac{3}{4tan13}$

5. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x + y$ with the initial condition $f(1) = 2$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x = 1$ with a step size of 0.5?

(A) 3

(B) 5

(C) 6

(D) 10

(E) 12

6. What are all values of p for which $\int_1^\infty \frac{1}{x^{2p}} dx$ converges?

(A) $\mathrm{p} < -1$

(B) $\mathrm{p} > 0$

(C) $\mathrm{p} > \frac{1}{2}$

(D) $\mathrm{p} > 1$

(E) There are no values of p for which this integral converges.

7. The position of a particle moving in the xy −plane is given by the parametric equations $x = t^3 - 3t^2$ and $y = 2t^3 - 3t^2 - 12t$. For what values of t is the particle at rest?

(A) $-1\ only$

(B) $0\ \ only$

(C) $2\ \ only$

(D) $-1\ and\ 2\ only$

(E) $-1, 0, and\ 2$

8. $\int x^2 \cos(x^3)\, dx =$

(A) $-\frac{1}{3}\sin(x^3) + C$

(B) $\frac{1}{3}\sin(x^3) + C$

(C) $-\frac{x^3}{3}\sin(x^3)$
$+ C$

(D) $\frac{x^3}{3}\sin(x^3) + C$

(E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right) + C$

9. If $f(x) = \ln{(x + 4 + e^{-3x})}$, then $f'(0)$ is

(A) $-\frac{2}{5}$

(B) $\frac{1}{5}$

(C) $\frac{1}{4}$

(D) $\frac{2}{5}$

(E) nonexistent

10. What is the value of $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$?

(A) 1

(B) 2

(C) 4

(D) 6

(E) The series diverges

11. The Maclaurin series for $\frac{1}{1-x}$ is $\sum_{n=0}^{\infty} x^n$. Which of the following is an power series expansion for $\frac{x^2}{1-x^2}$?

(A) $1 + x^2 + x^4 + x^6 + x^8 + \cdots \ldots \ldots$

(B) $x^2 + x^3 + x^4 + x^5 + \cdots \ldots \ldots$

(C) $x^2 + 2x^3 + 3x^4 + 4x^5 + \cdots \ldots \ldots$

(D) $x^2 + x^4 + x^6 + x^8 + \cdots \ldots \ldots$

(E) $x^2 - x^4 + x^6 - x^8 + \cdots \ldots \ldots$

12. The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

(A) $\mathrm{V(t)} = \mathrm{k}\sqrt{t}$

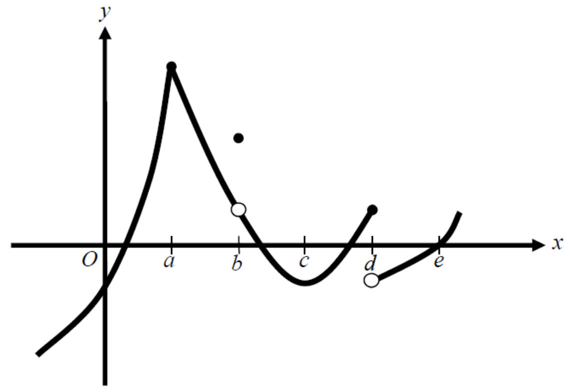
(B) $\mathrm{V(t)} = \mathrm{k}\sqrt{V}$

(C) $\frac{\mathrm{dV}}{dt} = \mathrm{k}\sqrt{t}$

(D) $\frac{\mathrm{dV}}{dt} = \frac{\mathrm{k}}{\sqrt{V}}$

(E) $\frac{\mathrm{dV}}{dt} = \mathrm{k}\sqrt{V}$

13. The graph of a function f is shown to the right. At which value of x is f continuous, but not differentiable?



(A) a

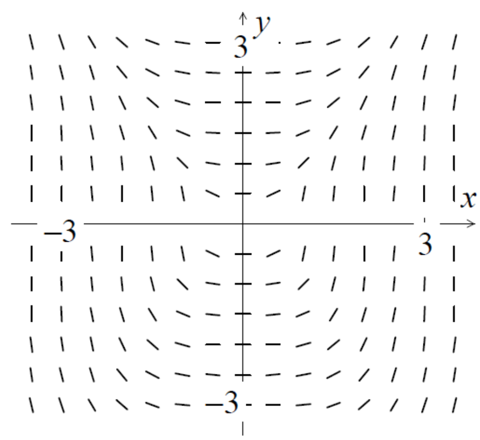
(B) b

(C) c

(D) d

(E) e

14. Shown at the right is a slope field for which of the differential equations?



(A) $\frac{dy}{dx} = \frac{x}{y}$

(B) $\frac{dy}{dx} = \frac{x^2}{y^2}$

(C) $\frac{dy}{dx} = \frac{x^3}{y}$

(D) $\frac{dy}{dx} = \frac{x^2}{y}$

(E) $\frac{dy}{dx} = \frac{x^3}{y^2}$

15. The length of a curve from $x = 1$ to $x = 4$ is given by $\int_1^4 \sqrt{1 + 9x^4}\, dx$. If the curve contains the point $(1,6)$, which of the following could be an equation for this curve?

(A) $y = 3 + 3x^2$

(B) $y = 5 + x^3$

(C) $y = 6 + x^3$

(D) $y = 6 - x^3$

(E) $y = \frac{16}{5} + x + \frac{9}{5}x^5$

16. If the line tangent to the graph of the function f at the point (1,7) passes through the point $(-2,-2)$, then $f'(1)$ is

(A) -5
(B) 1
(C) 3
(D) 7
(E) undefined

17. A curve C is defined by the parametric equations $x = t^2 - 4t + 1$ and $y = t^3$. Which of the following is an equation of the line tangent to the graph of C at the point $(-3,8)$?

(A) $x = -3$

(B) $x = 2$

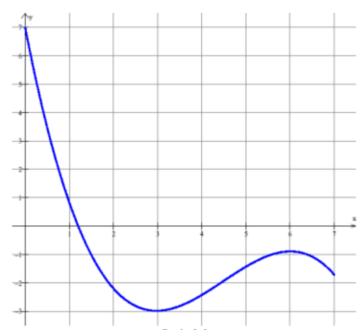
(C) $y = 8$

(D) $y = -\frac{27}{10}(x+3) + 8$

(E) $y = 12(x+3) + 8$

18. The graph of the function f shown in the figure has horizontal tangents at $x = 3$ and $x = 6$. If $g(x) = \int_0^{2x} f(t)dt$, what is the value of $g'(3)$?

(A) 0
(B) -1
(C) -2
(D) -3
(E) -6


Graph of f

19. A curve has slope $2x + 3$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1,2)$?

(A) $y = 5x - 3$

(B) $y = x^2 + 1$

(C) $y = x^2 + 3x$

(D) $y = x^2 + 3x - 2$

(E) $y = 2x^2 + 3x - 3$

20. A function f has Maclaurin series given by $\frac{x^4}{2!}+\frac{x^5}{3!}+\frac{x^6}{4!}+\cdots+\frac{x^{n+3}}{(n+1)!}$
Which of the following is an expression for $f(x)$?

(A) $-3xsin(x)+3x^2$

(B) $-\cos(x^2)+1$

(C) $-x^2\cos(x)+x^2$

(D) $x^2e^x-x^3-x^2$

(E) $e^{x^2}-x^2-1$

21. The number of moose in a national park is modeled by the function M that satisfies the logistic differential equation $\frac{dM}{dt}=0.6M(1-\frac{M}{200})$, where t is the time in years and $M(0)=50$. What is $\lim_{t\to\infty} M(t)$?

(A) 50

(B) 200

(C) 500

(D) 1000

(E) 2000

22. What are all values of p for which the infinite series $\sum_{n=1}^{\infty}\frac{n}{n^p+1}$ converges?

(A) $\mathrm{p}>0$

(B) $\mathrm{p}\geq 1$

(C) $\mathrm{p}>1$

(D) $\mathrm{p}\geq 2$

(E) $\mathrm{p}>2$

23. $\int x\sin(6x)dx =$

(A) $x\cos(6x) + \sin(6x) + C$

(B) $-\frac{x}{6}\cos(6x) + \frac{1}{36}\sin(6x) + C$

(C) $-\frac{x}{6}\cos(6x) + \frac{1}{6}\sin(6x) + C$

(D) $\frac{x}{6}\cos(6x) + \frac{1}{36}\sin(6x) + C$

(E) $6x\cos(6x) - \sin(6x) + C$

24. Which of the following series diverge?

I. $\sum_{n=0}^{\infty}\left(\frac{\sin 2}{\pi}\right)^n$

II. $\sum_{n=0}^{\infty}\frac{1}{\sqrt[3]{n}}$

III. $\sum_{n=0}^{\infty}\left(\frac{e^n}{e^n+1}\right)$

(A) III *only*

(B) I *and* II *only*

(C) I *and* III *only*

(D) II *and* III *only*

(E) I, II, *and* III

25. The function f is continuous on the closed interval $[2,14]$ and has values shown in the table. Using the subintervals $[2,5]$, $[5,10]$, and $[10,14]$, what is the approximation of $\int_2^{14} f(x)dx$ found using a right Riemann sum?

x	2	5	10	14
$f(x)$	12	28	34	30

(A) 296

(B) 312

(C) 343

(D) 374

(E) 390

26. $\int \frac{2x}{(x+2)(x+1)} dx =$

(A) $\ln|x+2| + ln|x+1| + C$

(B) $\ln|x+2| + ln|x+1| - 3x + C$

(C) $-4\ln|x+2| + 2ln|x+1| + C$

(D) $4\ln|x+2| - 2ln|x+1| + C$

(E) $2\ln|x| + \frac{2}{3}x + \frac{1}{2}x^2 + C$

27. $\frac{d}{dx}\left(\int_0^{x^3} ln(t^2+1)dt\right) =$

(A) $\dfrac{2x^3}{x^6+1}$

(B) $\dfrac{3x^2}{x^6+1}$

(C) $\ln(x^6+1)$

(D) $2x^3\ln(x^6+1)$

(E) $3x^2\ln(x^6+1)$

28. What is the coefficient of x^2 in the Taylor series for $\frac{1}{(1+x)^2}$ about $x = 0$?

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) 1

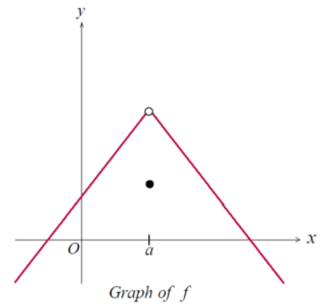
(D) 3

(E) 6

Multiple Choice #4
Calculator

Name____________________
Date____________Period___

1. The graph of the function f is shown. Which of the following statements must be false?



(A) $f(a) exists$

(B) $f(x)\ is\ defined\ for\ 0 < x < a$

(C) f is not continuous at $x = a$

(D) $\lim_{x\to a} f(x)\ exists$

(E) $\lim_{x\to a} f'(x)\ exists$

2. Let $P(x) = 3x^2 - 5x^3 + 7x^4 + 3x^5$ be the fifth-degree Taylor polynomial for the function f about $x = 0$. What is the value of $f'''(0)$?

(A) -30

(B) -15

(C) -5

(D) $-\frac{5}{6}$

(E) $-\frac{1}{6}$

3. The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

(A) $0.04\pi\ m^2/sec$

(B) $0.4\pi\ m^2/sec$

(C) $4\pi\ m^2/sec$

(D) $20\pi\ m^2/sec$

(E) $100\pi\ m^2/sec$

4. The table gives values of $f, f', g, and\ g'$ at selected values of x. If $h(x) = f(g(x))$, then $h'(1) =$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	6	5	3	-2
1	3	-3	-1	2
3	1	-2	2	3

(A) 5

(B) 6

(C) 9

(D) 10

(E) 12

5. Insects destroyed a crop at the rate of $\frac{100e^{-0.1t}}{2-e^{-3t}}$ tons per day, where time t is measured in days. To the nearest ton, how many tons did the insects destroy during the time interval $7 \le t \le 14$?

(A) 125

(B) 100

(C) 88

(D) 50

(E) 12

6. The graph of the function f is shown in the figure. The value of $\lim_{x\to 1} \sin(f(x))$ is

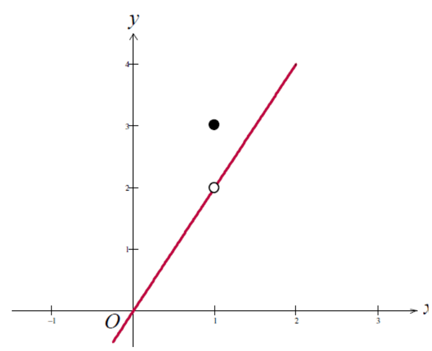
(A) 0.909

(B) 0.841

(C) 0.141

(D) -0.416

(E) nonexistent



7. The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

(A) $\int_{1.572}^{3.514} r(t)dt$

(B) $\int_{0}^{8} r(t)dt$

(C) $\int_{0}^{2.667} r(t)dt$

(D) $\int_{1.572}^{3.514} r'(t)dt$

(E) $\int_{0}^{2.667} r'(t)dt$

8. The function f is continuous and differentiable on the closed interval$[0,4]$. The table gives selected values of f on this interval. Which of the following statements must be true?

x	0	1	2	3	4
$f(x)$	2	3	4	3	2

(A) The minimum value of f on $[0,4]$ is 2.

(B) The maximum value of f on $[0,4]$ is 4.

(C) $f(x) > 0 \ for \ 0 < x < 4$

(D) $f\,'(x) < 0 \ for \ 2 < x < 4$

(E) There exists c, with $0 < c < 4, for\ which\ f'(c) = 0$

9. A particle moves in the xy –plane so that its position at any time t is given by $x(t) = t^2$ and $y(t) = \sin(4t)$. What is the speed of the particle when $t = 3$?

(A) 2.909

(B) 3.062

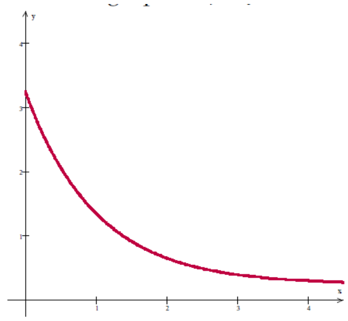
(C) 6.884

(D) 9.016

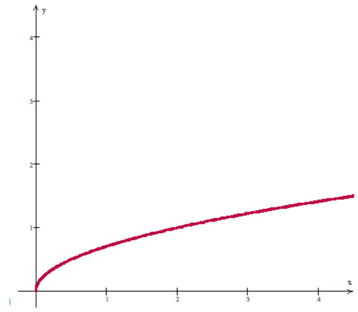
(E) 47.393

10. If a trapezoidal sum over approximates $\int_0^4 f(x)dx$ and a right Riemann sum under approximates $\int_0^4 f(x)dx$, which of the following could be the graph of $y = f(x)$?

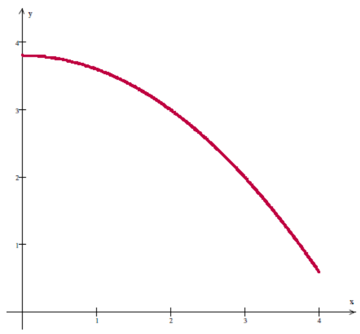
(A)



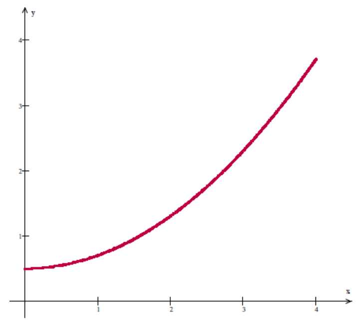
(B)



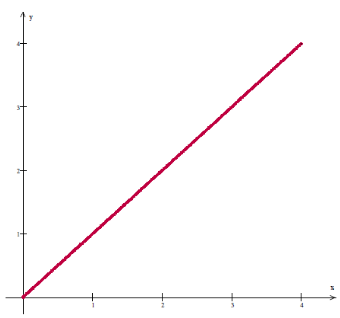
(C)



(D)



(E)



11. Let f be the function with derivative defined by $f'(x) = \sin(x^3)$ on the interval $-1.8 < x < 1.8$. How many points of inflection does the graph of f have on this interval?

(A) Two

(B) *Three*

(C) Four

(D) Five

(E) Six

12. A particle moves along the x −axis so that at any time $t \geq 0$, its velocity is given by $v(t) = \cos(2 - t^2)$. The position of the particle is 3 at time $t = 0$. What is the position of the particle when its velocity is first equal to 0?

(A) 0.411

(B) 1.310

(C) 2.816

(D) 3.091

(E) 3.411

13. The region bounded by the graph of $y = 2x - x^2$ and the x −axis is the base of a solid. For this solid, each cross-section perpendicular to the x −axis is an equilateral triangle. What is the volume of this solid?

(A) 1.333

(B) 1.067

(C) 0.577

(D) 0.462

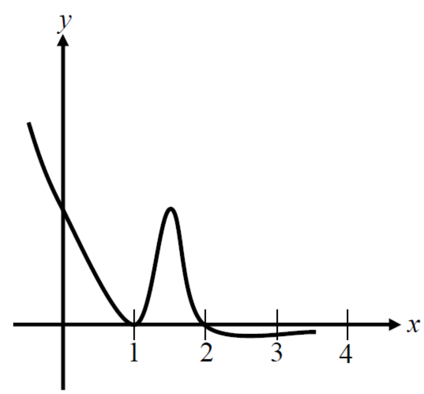
(E) 0.267

14. The graph of f', the derivative of the function f, is shown. If $f(0) = 0$, which of the following must be true?

I. $f(0) > f(1)$

II. $f(2) > f(1)$

III. $f(1) > f(3)$



(A) I *only*

(B) II *only*

(C) III *only*

(D) I *and* II *only*

(E) II *and* III *only*

15. The height$h$, in meters, of an object at time t is given by $h(t) = 24t + 24t^{3/2} - 16t^2$. What is the height of the object at the instant when it reaches its maximum upward velocity?

(A) 2.545 meters

(B) 10.263 meters

(C) 34.125 meters

(D) 54.889 meters

(E) 89.005 meters

16. Let f be the function defined by $f(x) = x + lnx$. What is the value of c for which the instantaneous rate of change of f at $x = c$ is the same as the average rate of change of f over $[1,4]$?

(A) 0.456

(B) 1.244

(C) 2.164

(D) 2.342

(E) 2.452

Multiple Choice #5 Name__________________
Non- Calculator Date___________Period___

1. $\int_0^5 \frac{dx}{\sqrt{3x+1}}$

(A) $\frac{1}{2}$

(B) $\frac{2}{3}$

(C) 1

(D) 2

(E) 6

2. Which of the following is continuous at $x=0$?

I. $f(x)=|x|$

II. $f(x)=e^x$

III. $f(x)=\ln\left(e^x-1\right)$

(A) *I only* (B) *II only* (C) *I and II only* (D) *II and III only* (E) *none*

3. For what value of c will $x^2+\frac{c}{x}$ have a relative minimum at $x=-1$?

(A) -4

(B) -2

(C) 2

(D) 4

(E) *None of these*

4. Which of the following could be a solution to the differential equation represented by the slope field given.

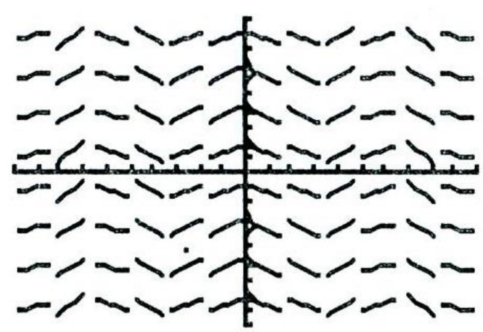
(A) $y=x^2$

(B) $y=\sin x$

(C) $y=\cos x$

(D) $y=e^x$

(E) $y=\ln x$



5. The volume of the solid generated by rotating about the $x-$axis the region enclosed between the curve $y=3x^2$ and the line $y=6x$ is given by

(A) $\pi\int_0^3\left(6x-3x^2\right)^2 dx$

(B) $\pi\int_0^2\left(6x-3x^2\right)^2 dx$

(C) $\pi\int_0^2\left(6x-36x^2\right)^2 dx$

(D) $\pi\int_0^2\left(36x^2-9x^4\right)dx\ e$

(E) $\pi\int_0^2\left(6x-3x^2\right)dx$

6. The asymptotes of the graph of the parametric equations $x=\frac{1}{t-1}$, $y=\frac{2}{t}$ are:

(A) $x=1,\ y=0$

(B) $y=2\ only$

(C) $x=-1,\ y=2$

(D) $x=-1\ only$

(E) $x=0,\ y=-1$

7. $\int_1^\infty \frac{3x^2}{\left(1+x^3\right)^2}dx=$

(A) $-\frac{1}{2}$

(B) 0

(C) $\frac{1}{2}$

(D) 1

(E) *nonexistent*

8. The radius of convergence of $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-2)^n}{n\bullet 3^n}$ is

(A) 3

(B) 2

(C) 1

(D) 0

(E) ∞

9. There is a point between $P(1,0)$ and $Q(e,1)$ on the graph of $y=\ln x$ such that the tangent to the graph at that point is parallel to the line through points P and Q. The $x-$coordinate of this point is

(A) $e-1$

(B) e

(C) -1

(D) $\frac{1}{e-1}$

(E) $\frac{1}{e+1}$

10. If $4x^2+2xy+3y=9$, then the value of $\frac{dy}{dx}$ at the point $(2,-1)$ is

(A) $-\frac{1}{2}$

(B) $\frac{1}{2}$

(C) 2

(D) -2

(E) *none of these*

11. If $\frac{dy}{dx}=\sqrt{x}$, then the average rate of change of y with respect to x on the closed interval $[0,4]$ is

(A) $\frac{1}{16}$ (B) 1 (C) $\frac{4}{3}$ (D) $\sqrt{2}$ (E) 2

12. A particle moves along the $x-$axis and its position for time $t\geq 0$ is $x(t)=\cos(2t)+\sec t$. When $t=\pi$, the acceleration of the particle is

(A) -6

(B) -5

(C) -4

(D) -3

(E) *none of these*

13. The region bounded by the $x-$axis and the part of the graph of $y=\sin x$ between $x=0$ and $x=\pi$ is separated into two regions by the line $x=p$. If the area of the region for $0\le x\le p$ exceeds the area of the region for $p\le x\le\pi$ by one square unit, then $p=$

(A) $\arccos\frac{1}{4}$

(B) $\arccos\frac{1}{3}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{3}$

(E) $\frac{2\pi}{3}$

14. The graph of $y=5x^4+3x^5$ has a point of inflection at

(A) $(0,0)$ *only*

(B) $(1,8)$ *only*

(C) $(-1,2)$ *only*

(D) $(0,0)$ *and* $(1,8)$

(E) $(0,0)$ *and* $(-1,2)$

15. If $h(x)=[f(x)]^2+f(x)g(x)$, $f'(x)=g(x)$ and $g'(x)=-f(x)$, then $h'(x)=$

(A) $f(x)g(x)$

(B) $2f(x)-f(x)g(x)$

(C) $[f(x)+g(x)]^2$

(D) $[f(x)-g(x)]^2$

(E) $[g(x)]^2+2g(x)f(x)-[f(x)]^2$

16. Which of the following integrals gives the length of the graph of $y = Arc\sin\frac{x}{2}$ between $x = a$ and $x = b$, where $0 \le a \le b \le \frac{\pi}{2}$?

(A) $\int_a^b \sqrt{1 - \frac{1}{\sqrt{4-x^2}}}\, dx$

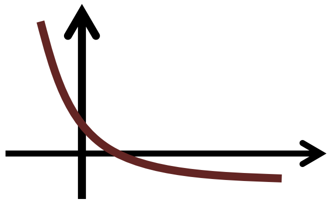
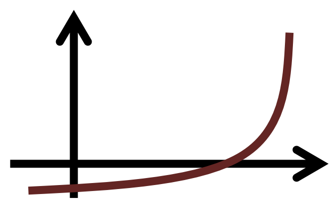
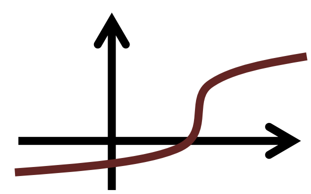
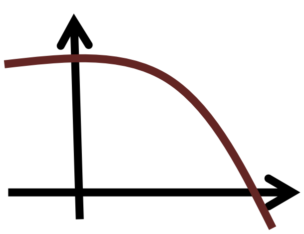
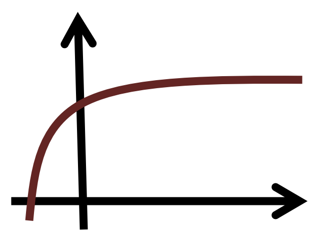
(B) $\int_a^b \sqrt{1 + \frac{1}{\sqrt{4-x^2}}}\, dx$

(C) $\int_a^b \sqrt{1 - \frac{1}{4-x^2}}\, dx$

(D) $\int_a^b \sqrt{1 + \frac{1}{4-x^2}}\, dx$

(E) $\int_a^b \left[1 + \frac{1}{4-x^2}\right] dx$

17. If y is a function of x such that $\frac{dy}{dx} > 0$ for all x and $\frac{d^2y}{dx^2} < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?

(A)		(B)		(C)	
(D)		(E)			

18. $\int \frac{1}{x^2+x}\,dx =$

(A) $\frac{1}{2}\arctan\left(x+\frac{1}{2}\right)+C$

(B) $\ln\left|x^2+x\right|+C$

(C) $\ln\left|\frac{x+1}{x}\right|+C$

(D) $\ln\left|\frac{x}{x+1}\right|+C$

(E) *none of these*

19. If $f(x)=x^2e^{-2x}$, then the graph of f is increasing for all x such that

(A) $0<x<1$

(B) $0<x<\frac{1}{2}$

(C) $0<x<2$

(D) $x<0$

(E) $x>0$

20. $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{n!}$ is the Taylor series about $x=0$ for which of the following functions?

(A) $\sin x$

(B) $\cos x$

(C) e^x

(D) e^{-x}

(E) $\ln(1+x)$

21. Evaluate $\int_{-\frac{\pi}{4}}^{e-1} f(x)dx$ if $f(x)=\begin{cases}\sec^2 x & for \quad x\le 0\\ \frac{1}{x+1} & for \quad x>0\end{cases}$

(A) 0

(B) 1

(C) 2

(D) e

(E) π

22. $\lim_{x\to 0}\left(1+\frac{x}{2}\right)^{\cot x} =$

(A) $\frac{e}{2}$

(B) e

(C) $\sqrt{e}$

(D) $2e$

(E) e^2

23. The area of the closed region bounded by the polar graph of $r=\sqrt{1+\cos\theta}$ is given by

(A) $\int_0^{2\pi}\sqrt{1+\cos\theta}\,d\theta$

(B) $\int_0^{\pi}\sqrt{1+\cos\theta}\,d\theta$

(C) $2\int_0^{2\pi}(1+\cos\theta)\,d\theta$

(D) $\int_0^{\pi}(1+\cos\theta)\,d\theta$

(E) $2\int_0^{\pi}\sqrt{1+\cos\theta}\,d\theta$

24. $\lim_{h\to 0}\left[\frac{(3+h)^5-3^5}{9h}\right]$ is

(A) 0

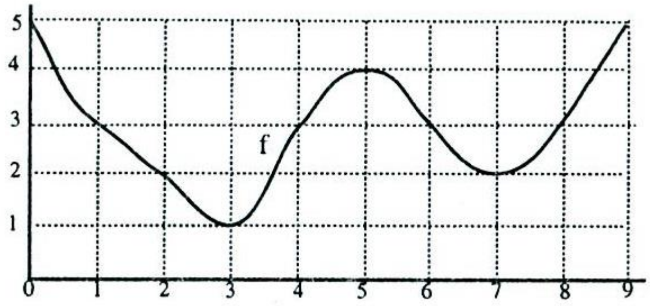
(B) 1

(C) 45

(D) 405

(E) *nonexistent*

25. Consider the function f whose graph is Shown at the right. Use the Trapezoid Rule with $n = 4$ to estimate the value of $\int_1^9 f(x)\,dx$



(A) 21

(B) 22

(C) 23

(D) 24

(E) 25

26. $\int x\sqrt{1-x^2}\,dx =$

(A) $\frac{(1-x^2)^{\frac{3}{2}}}{3}+C$

(B) $-(1-x^2)^{\frac{3}{2}}+C$

(C) $\frac{x^2(1-x^2)^{\frac{3}{2}}}{3}+C$

(D) $-\frac{x^2(1-x^2)^{\frac{3}{2}}}{3}+C$

(E) $-\frac{(1-x^2)^{\frac{3}{2}}}{3}+C$

27. Suppose a continuous function f and its derivative f' have values as given in the following table. Given that $f(1)=2$, use Euler's method to approximate the value of $f(2)$.

x	1.0	1.5	2.0
$f'(x)$	0.4	0.6	0.8
$f(x)$	2.0		

(A) 2.1

(B) 2.3

(C) 2.5

(D) 2.7

(E) 2.9

28. A particle is moving in the xy–plane and its position at time t is given by $x=\cos\left(\frac{\pi}{3}t\right)$ and $y=2\sin\left(\frac{\pi}{3}t\right)$. When $t=3$, the speed of the particle is

(A) $-\frac{2\pi}{3}$ (B) 0 (C) $\frac{2\pi}{3}$ (D) π (E) $\frac{\sqrt{13}}{3}\pi$

Multiple Choice #6 Name___________________
Calculator Date____________Period___

1. The slope of the curve $y = x^2 - e^{-x}$ at its point of inflection is

(A) $-\ln 2$ (B) $-\ln 4$ (C) $2-\ln 4$ (D) $2+\ln 4$ (E) $\frac{e^2}{2}$

2. The graph of the parabola $y = 2x^2 + x + k$ is tangent to the line $3x + y = 1$, then $k =$

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

3. If a function f is defined by $f(x) = \int_0^x \frac{1}{1+t^4}\,dt$, which of the following statements are true?

I. $f(1) = \frac{1}{2}$

II. the graph of f is concave down at $x = 3$

III. $f(x) + f(-x) = 0$ for all real numbers x

(A) I $only$ (B) II $only$ (C) III $only$ (D) II and III $only$ (E) I, II, III

4. Consider the function f defined on the domain $-0.5 \le x \le 0.5$ with $f(0) = 1$, and $\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \sec^2(3x)$. Evaluate: $\int_0^{0.5} f(x)\,dx$.

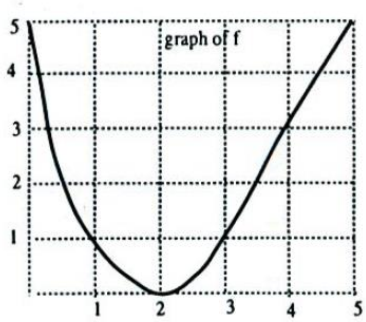
(A) 0.294 (B) 0.794 (C) 1.294 (D) 1.794 (E) 4.700

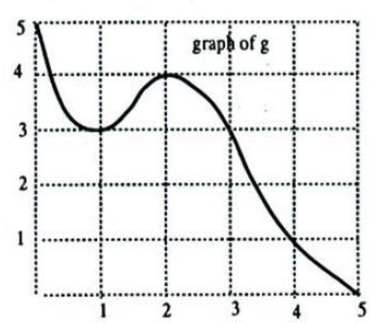
5. The graphs of the functions f and g are shown at the right.
If $h(x) = g[f(x)]$, which of the following statements are true about tl function h?

I. $h(2) = 5$.

IV. is increasing at $x = 4$.

V. The graph of h has a horizontal tangent at $x = 1$.





(A) I $only$ (B) II $only$ (C) III $only$ (D) I and III $only$ (E) I, II, III

6. A particle moves along the $x-$axis so that at any time $t \geq 0$ its velocity is given by $v(t) = \cos\left(t + \sqrt{t}\right)$. The total distance traveled by the particle from $t = 0$ to $t = 4$ is

(A) 2.26 (B) 2.30 (C) 2.34 (D) 2.38 (E) 2.42

7. The area of the region bounded by the graphs of $y = \arctan x$ and $y = 4 - x^2$ is approximately

(A) 10.80 (B) 10.97 (C) 11.14 (D) 11.31 (E) 11.48

8. Which of the following series are conditionally convergent?

I. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n+1}$ II. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\cos n}{3^n}$ III. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$

(A) *I only* (B) *II only* (C) *I and II only* (D) *I and III only* (E) *I, II, III*

9. If $x = t^2$ and $y = \ln\left(t^2 + 1\right)$, then at $t = 1$, $\frac{d^2y}{dx^2}$ is

(A) $-\frac{1}{4}$ (B) $-\frac{1}{2}$ (C) -1 (D) 0 (E) $\frac{1}{4}$

10. When a wholesale produce market has x crates of lettuce available on a given day, it charges p dollars per crate as determined by the supply equation $px - 20p - 6x + 40 = 0$. If the daily supply is decreasing at the rate of 8 crates per day, at what rate is the price changing when the supply is 100 crates?

(A) *not changing*

(B) *inc. at* \$0.10 *per day*

(C) *dec. at* \$0.10 *per day*

(D) *inc. at* \$1.00 *per day*

(E) *dec. at* \$1.00 *per day*

11. Let R be the region in the first quadrant bounded above by the graph of $f(x) = 2Arc\tan x$ and below by the graph of $y = x$. What is the volume of the solid generated when R is rotated about the $x-$axis?

(A) 1.21 (B) 2.28 (C) 2.69 (D) 6.66 (E) 7.15

12. For any time $t \geq 0$, if the position of a particle is the $xy-$plane is given by $x = e^t$ and $y = e^{-t}$, then the speed of the particle at time $t = 1$ is

(A) 2.693 (B) 2.743 (C) 3.086 (D) 3.844 (E) 7.542

13. If the derivative of the function f is $f'(x) = 3(x+2)(x+1)^2(x-3)^3$, then f has a local minimum at $x =$

(A) -2 *only* (B) -1 *only* (C) 3 *only* (D) -2 *and* 3 (E) -1 *and* 3

14. What is the $x-$coordinate of the point on the curve $y = e^x$ that is closest to the origin?

(A) -0.452 (B) -0.426 (C) -0.400 (D) -0.374 (E) -0.372

15. Let R be the region in the first quadrant enclosed by the graphs of $y = x\cos x$, $x = 0$, and $x = k$ for $0 < k < \frac{\pi}{2}$. The area of R, in terms of k, is

(A) $k\sin k + \cos k - 1$

(B) $-\cos k + \sin k$

(C) $-k\sin k + \cos k - 1$

(D) $k\sin k - \cos k + 1$

(E) $-k\sin k - \cos k + 1$

16. If $\int f(x)\cos x\, dx = f(x)\sin x - \int 6x^2 \sin x\, dx$, then $f(x)$ could be

(A) $-2x^3$

(B) $2x^3$

(C) $-3x^2$

(D) $3x^2$